MATHEMATICS OF BUSINESS AND COMMERCE

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MATHEMATICS OF BUSINESS & COMMERCE

BY
O. H. COCKS AND E. P. GLOVER

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INTRODUCTION

In compiling this book the requirements of the average student of the Commercial section of the Continuation Schools have been constantly kept in view. Commerce provides such a wide range of mathematical problems that it has been considered unnecessary to introduce examples which have no commercial value.

The admirable syllabus of the Royal Society of Arts has been taken as the basis of the book, and the student who works through the examples may face with confidence any problem likely to arise in the course of business.

As the student of the Continuation School has already passed through the Primary Schools, a knowledge of the more elementary arithmetical processes has been assumed. These processes have therefore only been referred to in order to suggest quicker methods of working. A chapter on Symbolical Expression has also been included for students without previous knowledge of algebraical representation, which will be found useful in solving some of the harder problems.

No formal examples on the use of four-figure logarithm tables have been included in Chapter XXIII, as the student should be encouraged to apply logarithmic calculations wherever possible to

the examples set throughout the volume. In this way he will be able to test both the utility and limitations of this form of calculation as applied to commercial problems.

O. H. C.
E. P. G.

July 1919.

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CHAPTER I

ADDITION AND SUBTRACTION

ADDITION

1. The ability to cast up long columns of figures with ease and rapidity is absolutely essential to students of commerce. It can only be acquired by constant practice, and, though there is no short way of acquiring speed and accuracy, the student will save time by adopting from the outset some such system as is suggested below.

In the first place all words and mental steps should be reduced to a minimum. Again, it is not always found most convenient to add numbers individually, or even in the order in which they occur. Any number which completes a ten with the figure in hand can be brought into the total some steps before its consecutive turn. Groups of small numbers also should be added into the sum as if they were one number.

2. These suggestions will be better understood by considering the steps given for the addition of the figures in the Balance Sheet below. This has been taken, with slight modification,

from a copy issued by a South African Gold Mining Co.

	£	s.	d.		£	s.	d.
Capital . .	325,000	0	0	Property .	205,884	4	6
Dividends payable . .	113,750	0	0	Machinery, etc.	112,477	5	1
Dividends unclaimed .	1,233	16	7	Native recruiting . .	1,113	13	7
Profits tax .	14,872	16	0	Share investments . .	3,370	8	9
Sundry creditors . .	7,372	15	9	Stores . .	4,486	5	8
Wages unpaid .	2,932	3	4	Sundry debtors	2,147	11	2
				Gold in transit	24,937	10	1
				Cash on deposit	142,823	4	8
Balance .	*48,924*	*9*	*5*	Cash in bank and mines .	16,845	17	7
	514,086	1	1		514,086	1	1

Casting up the pence in the right-hand column, the steps are: 7—16—26—36—43—49 = 4/1. Put down 1 in the pence.

Carry 4 to the shillings—11—15—20—29—32—37—4*1*. Put down 1 in the shillings.

Carry 4—5—6—7—8.

Carry 4 to the £'s—9—19—26—32—42—4*6*. Put down 6.

Carry 4—10—17—25—32—40—4*8*. Put down 8.

Carry 4—12—20—30—37—42—5*0*. Put down 0.

Carry 5—11—13—23—27—29—3*4*. Put down 4.

Carry 3—4—10—1*1*. Put down 1.

Carry 1—2—3—*5*. Put down 5.

The student should cast up the above for himself and endeavour to see the reason for the steps given. With practice he will find they follow much more naturally and easily than addition of consecutive figures. The grouping will probably vary with different individuals, but that given above will indicate sufficiently methods by which the work can be abbreviated.

EXAMPLES Ia

No. 1

Find the totals to the following columns, (a), (b), *and* (c):

RAILWAY STATISTICS FOR YEAR ENDING DECEMBER 31, 1917.

Railway	(a) Total Expenditure on Capital A/c.	(b) Gross Receipts	(c) Working Expenses	(d) % on Gross Receipts	(e) Net Receipts
	£	£	£		
(i) Barry .	6,300,371	1,050,526	703,415		
(ii) Cambrian .	6,478,759	421,224	282,137		
(iii) Cent. Lond.	4,548,681	351,289	192,465		
(iv) City and Cent.Lond.	3,423,829	258,233	141,185		
(v) Furness .	7,042,900	894,980	629,959		
(vi) Gt. Central	57,236,975	7,832,481	5,728,171		
(vii) Gt. Eastern	54,074,318	7,842,106	5,807,918		
(viii) Gt.Northrn.	54,655,647	8,269,245	6,047,501		
(ix) Gt. Western	114,699,826	18,810,744	13,210,438		
(x) Hull and Barnsley .	10,094,115	973,321	577,255		
(xi) Lancs. and Yorks .	65,211,137	9,052,810	6,626,881		
(xii) L. & N.-W.	124,640,110	21,484,097	15,587,384		
(xiii) L. & S.-W	50,890,811	7,655,801	5,620,504		
(xiv) L., Brighton, and S.Coast	33,279,543	4,591,888	3,206,548		
(xv) L., Chathm., and Dover	24,400,970	108,068	109,213		
(xvi) Lond. Elect.	17,869,416	1,139,554	611,892		
(xvii) Maryport & Carlisle .	920,279	147,048	95,045		
(xviii) Metropoltn.	18,138,238	1,166,414	745,928		
(xix) Metropoltn. District .	11,571,837	1,122,068	697,433		
(xx) Midland .	130,175,167	18,167,160	12,514,686		
(xxi) N.-Eastern	86,496,133	14,034,270	10,000,245		
(xxii) N. London	4,200,888	502,354	357,456		
(xxiii) N. Staff. .	9,127,796	1,381,707	958,535		
(xxiv) Rhymney .	2,403,707	439,792	284,366		
(xxv) S.-Eastern	33,732,414	163,153	154,136		
(xxvi) S.-E. and Chatham .	4,573,208	6,059,661	4,057,145		
(xxvii) Taff Vale .	6,652,837	1,321,262	885,084		
Totals .					

No. 2

The numbers of passengers carried by certain railways during the year 1909 were as follows :

	First Class	Second Class	Third Class	Total
(a)	1,623,307	2,705,657	96,012,958	
(b)	1,464,177	2,348,040	88,833,676	
(c)	1,315,085	4,206,190	75,416,100	
(d)	2,801,980	172,126	73,693,691	
(e)	1,951,958	2,818,097	59,740,357	
(f)	1,118,995	3,202,093	59,514,689	
(g)	1,592,450	4,001,366	51,553,035	
(h)	1,279,525	3,460,429	43,370,939	
(i)	731,576	628,003	35,803,685	
(j)	2,053,203	4,492,711	29,246,329	

Fill in the totals for each line, and then cast up the four columns. Check the totals at the foot of the first three columns by that at the foot of the fourth column.

SUBTRACTION

3. In subtraction we endeavour to find how much larger one number is than another; or in other words, to find a number which, added to the smaller, makes it equal to the larger.

The usual methods of finding the difference between two numbers adopt the plan of *taking away* the lesser from the larger. The method of *complementary addition*, however, *adds* a number to the lesser one, so that their total equals the larger number. This method is capable of giving quick results, and is also useful from the fact

that its principle is of use when balancing long columns of figures.

4. The adjoining examples will make its working clear.

(a) *Find the difference between* 8,521 *and* 5,843.

5,843
3,521
2,322

In working we seek to find numbers which added to 3, 5, 2, 1 give 5, 8, 4, 3 respectively. Commencing at the units steps by step, we get:

1 and *2* make 3. Set down 2.
2 and *2* make 4. Set down 2.
5 and *3* make 8. Set down 3.
3 and *2* make 5. Set down 2.

When some of the lower digits are larger than those immediately above as in the second example: *e.g.* in the case of the units 5 and 1, the number sought will not add up with the 5 to make 1, but will give 1*1*. This gives a 1 in the units column in just the same manner, but also gives 1 to carry forward.

15,321
3,795
11,526

The following are the steps in this case:
5 and *6* make 11. Set down 6, carry 1.
9 + 1 are 10 and *2* make 12. Set down 6, carry 1.
7 + 1 are 8 and *5* make 13. Set down 5, carry 1.
3 + 1 are 4 and *11* make 15. Set down 11.

5. In compound subtraction the same processes are used. Cf. the following example:

Find the difference between £36,491 4*s.* 5¼*d. and* £8,974 12*s.* 4½*d.*

£	*s.*	*d.*
36,491	4	5¼
8,974	12	4½
27,516	12	0¾

2

Farthings :
2 and *3* make 5. Set down 3, carry 1*d.*
Pence :
4 + 1 are 5 and *0* make 5. Set down 0.
Shillings :
12 and *12* are 24. Set down 12, carry £1.
Pounds :
4 + 1 are 5 and *6* make 11. Set down 6, carry 1.
7 + 1 are 8 and *1* make 9. Set down 1.
9 and *5* are 14. Set down 5, carry 1.
8 + 1 are 9 and *7* make 16. Set down 7, carry 1.
1 and *2* make 3. Set down 2.

Examples I*b*

Fill in column (*c*) of Example I by finding the difference between the Working Expenses and Gross Receipts, using the above method of subtraction.

6. The use of *Complementary Addition* in balancing a column of figures can be seen with reference to the example given in Para. 2, where it is required to fill in the balance item of the left-hand side.

Instead of casting up the columns, setting down the total and subtracting from the right-hand side, the correct sum as given on the right is filled in at the bottom, and the balance item completed figure by figure—by casting up each column and setting down that digit which is required to bring the total into agreement with that given at the foot of the column. Thus commencing with the pence :

4—13—20 and *5* make 25. Set down 5*d.*, carry 2/-.

2—5—10—22 and *9* make 31. Set down 9, carry 3.

3—4—5—6 and *0* make 60. Set down —, carry £3.

3—9—12 and *4* make 16. Set down 4, carry 1.

4—11—21—26 and *2* make 28. Set down 2, carry 2.

11—14—24—31 and *9* make 40. Set down 9, carry 4.

6—13—18—21—26 and *8* make 34. Set down 8, carry 3.

3—4—5—7 and *4* make 11. Set down 4, carry 1.

1—2—5. No further number required.

The figures supplied are therefore £48,924 9*s.* 5*d.*

EXAMPLES Ic

In each of the following add up the right-hand columns and supply the totals. Write this at the foot of the left-hand column, and then supply the balance needed as in the above manner.

No. 1

	£	*s.*	*d.*	£	*s.*	*d.*
	16,268	1	2	13,297	2	5
	1,686	13	8	4,658	1	3
	2,301	13	2	2,785	12	9
	1,659	10	9	5,483	7	11
	767	1	4	1,843	9	3
	301	3	3	786	11	2
	600	0	0	1,943	2	5
	781	2	9	856	3	8
Balance				2,497	5	1

No. 2

	£	£
	22,589	353,980
	223,413	221,729
	40,441	292,895
	424,106	849,853
	481,392	301,727
	235,538	16,498
	314,678	123,311
		68,429
		7,431
Balance		

No. 3

£	s.	d.	£	s.	d.
200,000	0	0	108,937	4	3
17,632	1	1	52,814	3	6
48,951	17	4	117,311	3	11
29,386	3	11	42,161	12	1
145	2	9	4,293	19	5
3,562	8	3	5,846	7	6
187	2	11	743	8	9
Balance			11,264	13	7

No. 4

£	£
749,632	243,782
118,542	151,685
29,684	297,342
32,741	411,629
483,152	728,311
69,784	121,149
73,291	17,832
	26,541
Balance	

No. 5

£	s.	d.	£	s.	d.
131,937	4	3	105,421	3	7
21,542	11	8	36,289	1	9
6,734	9	5	27,482	4	3
2,434	8	1	7,415	12	11
1,765	3	11	832	7	1
2,842	19	7	6,421	8	3
934	13	8	741	11	4
			832	5	1
Balance			606	17	9

No. 6

£	£
493,621	214,856
19,851	148,732
134,297	106,297
65,841	14,215
232,859	149,732
164,329	185,392
71,488	241,672
9,544	18,359
Balance	164,295

The student can obtain further exercises in the above work by clipping from the finance columns of newspapers the balance sheets often published by banks and commercial houses. The totals and balances can be covered with a sheet of paper, the columns cast up and balanced, and then compared with the figures actually given.

CHAPTER II

SHORT METHODS OF MULTIPLICATION AND DIVISION

MULTIPLICATION

7. WHEN two or more numbers are multiplied together the result is called the *Product* of the given numbers, and each of the numbers is called a *Factor* of the product. e.g. $5 \times 7 \times 8 \times 10 =$ 2,800. 5, 7, 8, and 10 are Factors of the Product 2,800.

8. If the factors are the same number, the product is called a *Power* of that number. Instead of writing the factors out in full, the number is written once only and a small figure known as an *Index* is placed above it, to show the number of times it is to be used as a factor, e.g.:

$100 = 10 \times 10$, usually written 10^2 and read 10 to the second power.

$1{,}000 = 10 \times 10 \times 10$, usually written 10^3 and read 10 to the third power.

$10{,}000 = 10 \times 10 \times 10 \times 10$, usually written 10^4 and read 10 to the fourth power.

10^2 and 10^3 are also referred to as 10 squared and 10 cubed respectively.

MENTAL RULES FOR MULTIPLICATION

9. (*a*) To multiply a number by 10 or any power of 10, move the decimal point one place to the right

for every cipher in the multiplier. In the case of whole numbers, ciphers are usually added instead of moving the decimal point.

e.g. 537·638×100 = 53763·8; 645×1,000 = 645,000.

(*b*) As direct deductions from the above rule we obtain short methods for multiplying by 5, 25, 125, 625, 50, 250, etc., etc.

To multiply by 5. Since $5 = \frac{10}{2}$, divide by 2 and multiply by 10.

To multiply by 25. Since $25 = \frac{100}{4}$, divide by 4 and multiply by 100.

To multiply by 125. Since $125 = \frac{1000}{8}$, divide by 8 and multiply by 1,000.

To multiply by 625. Since $625 = \frac{10000}{16}$, divide by 16 and multiply by 10,000.

e.g. (*a*) *Multiply* 237 *by* 25.—237 divided by 4 gives 59 and 1 over. There is no need to divide further, since the 1 over gives 25 as the last two figures. Final product = 5,925.

(*b*) *Multiply* 1,867 *by* 125.—1,867 divided by 8 gives 233 and 3 over. As before, no further division is necessary, since the last three figures 375 can be written down from the 3 immediately. Final product = 233,375.

Note.—The last two figures in (*a*) for remainders 1, 2, 3 are 25, 50, 75 respectively, while the last three figures in (*b*) for remainders 1 to 7 are 125, 250, 375, 500, 625, 750, and 875 respectively.

Miscellaneous Multipliers

10. Quick methods of multiplication for certain other multipliers can be obtained by basing them upon those given above, e.g. numbers near 1,000 can be based upon 1,000, those near 625 upon 625,

and so on. The following examples illustrate the method :

(*a*) 1,649 × 253

412,250	=	250	times	1,649
4,947	=	3	,,	1,649
417,197				

(*b*) 1,729 × 998

1,729,000	=	1,000	times	1,729
3,458	=	2	,,	1,729
1,725,542	=	998	,,	1,729

(*c*) 3,485 × 127

435,625	=	125	times	3,485
6,970	=	2	,,	3,485
442,595	=	127	,,	3,485

11. Certain combinations of figures in the multiplier often admit of a considerable shortening in the work, e.g. :

(*a*) Multiply 43,972 by 729.

Note : 729 = 720 + 9
= 80 nines + 9

43,972	
729	
395,748	= 9 times 43,972
31,659,84	= 80 times above line
32,055,588	

(*b*) Multiply 13,489 by 13,212

13,212 = 13,200 + 12
= 1,100 twelves + 12

13,489	
13,212	
161,868	= 12 times 13,489
178,054,8	= 1,100 times above line
178,216,668	

12. *To multiply in one line by any number between* 12 *and* 100.

Multiply 6,482 by 37. Worked by the ordinary methods this would be set out as shown. If we examine the manner in which the final product is built up, we see that the work could have been performed quite as easily, and with a saving of time and space, by applying the following rule :

```
  6,482
     37
 45,374
194,46
239,834
```

Multiply each figure of the top line by the units figure of the multiplier, and the figure to the right of this one by the tens figure, add the results together mentally, set down and carry in the usual manner.

The above example worked by the above method would be as follows :

```
   6482
     37
239,834
```

Note : It is advisable to tick each figure of the top line as it is multiplied by the units so that there can be no mistake in picking up the next figure to be multiplied.

(1)	$2 \times 7 = 14$	Set down *4*. Carry 1.
(2)	$8 \times 7 = 56$ and $1 = 57$	
	$2 \times 3 = 6$ and $57 = 63$	Set down *3*. Carry 6.
(3)	$4 \times 7 = 28$ and $6 = 34$	
	$8 \times 3 = 24$ and $34 = 58$	Set down *8*. Carry 5.
(4)	$6 \times 7 = 42$ and $5 = 47$	
	$4 \times 3 = 12$ and $47 = 59$	Set down *9*. Carry 5.
(5)	$6 \times 3 = 18$ and $5 = 23$	Set down *23*.

DIVISION

13. A number is divided by any power of ten, if its decimal point is moved as many places to the left as there are ciphers in the divisor.

e.g. 637 ÷ 1,000 = ·637   63·7 ÷ 10,000 = ·00637

14. Methods of dividing by 5, 25, 125, 625, etc., are derived from the above as follows :

To divide by 5. Since $5 = \frac{10}{2}$, first multiply by 2, then divide by 10.

To divide by 25. Since $25 = \frac{100}{4}$, first multiply by 4, then divide by 100.

To divide by 125. Since $125 = \frac{1000}{8}$, first multiply by 8, then divide by 1,000.

To divide by 625. Since $625 = \frac{10000}{16}$, first multiply by 16, then divide by 10,000.

e.g. $3{\cdot}743 \div 125 = {\cdot}029944$

Ignoring the decimal point and multiplying by 8 we get 29,944. There were three decimal figures in the original number, dividing by 1,000 will give three more, or six altogether, and counting these off from the above we get, final result, ·029944.

15. *Italian Method of Division.*—The Italian method differs from the usual method of "long division" only in the respect that the multiplications and subtractions are performed in one line, thus saving time and space, e.g. :

Divide 294,331 by 67.

(*a*) *Long Division.*

```
67)294,331(4,393
   268
   ---
    263
    201
    ---
     623
     603
     ---
      201
      201
      ---
      . . .
```

(*b*) *Italian Method.*

```
67)294,331(4,393
     263
      623
       201
       . . .
```

In (*b*) divide as in (*a*), but instead of setting down the product 268 and subtracting as a whole, per-

from the subtraction as each figure of the divisor is multiplied—using the method of complementary addition.

The first remainder 26 is obtained as follows:

$7 \times 4 = 28$ and *6* $= 34$. Set down *6*. Carry 3.
$6 \times 4 = 24$ and 3 carried $= 27$ and *2* $= 29$.
Set down *2*.

The other remainders are obtained in the same manner.

Example (*c*). *Divide* 6,482,775 *by* 2,379 (*Italian Method*).

```
2,379)6482775(2,725
        17247
         5947
         11895
         .....
```

The first remainder 1,724 is obtained as follows:

$9 \times 2 = 18$ and *4* $= 22$. Set down *4*. Carry 2.
$7 \times 2 = 14$ and 2 carried $= 16$ and *2* $= 18$. Set down *2*. Carry 1.

$3 \times 2 = 6$ and 1 carried $= 7$ and *7* $= 14$. Set down *7*. Carry 1.

$2 \times 2 = 4$ and 1 carried $= 5$ and *1* $= 6$. Set down *1*.

16. When both divisor and dividend are large numbers it is found convenient to arrange the divisor so that there is only one figure to the left of the decimal point. If the decimal point is then moved the same number of places in the dividend, the quotient of the two will be unaltered by the rearrangement.

Since the first figure of the divisor is now expressed in units, each figure of the dividend will

when divided give a figure of corresponding value in the quotient, e.g. the hundreds will give hundreds, tens give tens, and so on. In writing down the quotient, therefore, we can place each of its figures above the one of corresponding value in the dividend, so that the decimal point of the quotient will be above that of the dividend.

Example (*a*). Divide 37,984,215 by 248,379 (to 3 places of decimals).

```
          152·928
2·48379)379·84215000
          131·4631
            7·27365
            2·306070
              706590
             2098320
              111288
```

Notes: (*a*) The decimal point is moved five places to the left in the divisor, therefore it must be moved the same number of places in the dividend. (*b*) Only the remainders are set down as by the Italian method.

When the decimal points are moved as above, a rough check on the answer is readily provided, e.g. the divisor lies between 2 and 3, so that the quotient lies between $\frac{380}{2}$ and $\frac{380}{3}$, i.e. between 190 and 120.

EXAMPLES II

In the examples of (3) and (4) it will be found advisable to work with the standard form, and count off the decimal places at the end.

				(*a*)	(*b*)	(*c*)	(*d*)
(1)	Multiply	63.895	by	5	25	125	625
(2)	,,	2.436	,,	50	250	125	6250
(3)	,,	1,983	,,	·5	2·5	12·5	6·25
(4)	,,	2,941	,,	·05	·25	1·25	·625
(5)	,,	3,148	,,	·005	·025	·0125	·0625

Work the following by contracted methods:

	(a)	(b)	(c)	(d)	(e)	(f)
(6) 6,384 ×	128	247	251	127	98.	102
(7) 3,497 ×	249	993	1252	627	99	123
(8) 1,923 ×	24·8	·251	1·253	6·29	·253	62·57

Work the following in one line:

	(a)	(b)	(c)	(d)	(e)
(9) 923 ×	19	17	18	15	13
(10) 7,184 ×	21	19	37	84	63
(11) 2,975 ×	13	27	31	48	56
(12) 1,428 ×	67	73	29	91	53

Work the following multiplication in two lines:

(13) 12,976 ×	968	567	9,911	1,089
(14) 38,421 ×	7,711	357	14,412	484
(15) 68,329 ×	12,525	50,025	5,125	325

(16), (17), (18), (19), (20). Work Nos. 1, 2, 3, 4, 5 as division instead of multiplication.

Work the following by the Italian method of division (to one place of decimals):

	(a)	(b)	(c)	(d)	(e)
(21) 39,852 ÷	69	73	39	54	27
(22) 256,834 ÷	219	178	195	186	227
(23) 374,829 ÷	317	1,483	2,971	6,389	7,145
(24) 294,326 ÷	4,152	3,973	8,539	2,647	1,832

Work the following in the manner shown in para. 16 (to one place of decimals):

	(a)	(b)	(c)
(25) 98,735,621 ÷	38,472	98,365	872,411
(26) 29,354,836 ÷	78,293	385,467	1,134,876

CHAPTER III

APPROXIMATION AND DECIMALISATION

APPROXIMATION

17. It is seldom in practice that quantities or measures can be expressed in figures with absolute accuracy, nor is it always desirable that they should be so expressed even if it were possible. It should therefore be recognised from the outset that most quantities are only approximately stated, with a greater or less degree of accuracy according to the manner in which the figures are to be used, or the methods by which they were obtained.

A person buying cloth usually estimates his requirements in yards, a plumber measures his work by the foot, while a person dealing in expensive material might quote his price by the inch. To the first person an error of one or two inches in several yards of cloth would be practically negligible, but if the last person dealt in platinum wire the same error would be a much more serious matter.

Again, in the purchase of 1 lb. of tea, a few ounces underweight would be considered a very large deficiency, yet the same error would never be detected in the purchase of 1 ton of coals. Similarly, while a clerk would consider £100 as a large item if it referred to an increase in his salary,

he would probably ignore so small a sum while reading the statistics relating to imports.

18. The above instances will possibly suggest to the student that in expressing quantities or measures in approximate form we should be governed not so much by the *Absolute Value* of the error committed, as by its *Relative Value*. In the former case the error is considered as an amount in itself, in the latter as a fractional part of the whole.

19. As examples of varying degrees of approximation and relative error, consider the following list of imports for the United Kingdom for the years 1913–17.

	(*a*)	(*b*)	(*c*)
1913	£768,734,739	£7,687 hundred thousands	£769 millions
1914	£696,635,113	£6,966 ,, ,,	£697 ,,
1915	£851,893,350	£8,519 ,, ,,	£852 ,,
1916	£948,506,492	£9,485 ,, ,,	£949 ,,
1917	£1,064,164,678	£10,642 ,, ,,	£1,064 ,,

The figures as actually published are given as in column (*a*). For most purposes, however, sufficient accuracy would have been obtained by writing as in column (*b*), correct to the nearest hundred thousand pounds, or even as in (*c*), correct to the nearest million pounds.

The statement "correct to the nearest hundred thousand pounds" will be understood by considering the figures for one particular year. Thus the figures for 1915 are £851,893,350. Since £8,518 hundred thousands differs from this amount by £93,350, while £8,519 hundred thousands differs from this amount by £6,650, the latter is seen to

be the *nearest* hundred thousand. This would also have been true if the omitted figures were any greater than £50,000. The *Relative Error* in this case $= \frac{8650}{851893350}$ approx. $= \frac{1}{110000}$. The same relative error made with regard to £1 would amount in absolute value to less than $\frac{1}{110}$ penny.

20. In general, when writing correct to any figure, we replace by ciphers all digits to its right, increasing it by unity if the first digit on the right is 5 or over 5.

21. The following examples illustrate the necessity for approximating when dealing with a long line of digits of small value.

(*a*) It would be useless to write down a debt as £3·638216, since all digits after the 8 have too small a value to be paid in any coin of the realm. If the amount be written as £3·638, the absolute value of the error is about $\frac{1}{5}$ of a farthing.

(*b*) For most practical purposes nothing is gained by writing the length of an object as 3·64781 feet. If this is written correct to two places, i.e. 3·65 feet, the error committed is about $\frac{1}{40}$ of an inch—or less than can be detected by ordinary methods.

EXAMPLES III*a*

(1) Write the figures in column (*a*) para. 19.

(*a*) Correct to the nearest 100, (*b*) to the nearest 1,000.

(2) Write the following correct to the third decimal place. What is the approximate relative error in each case?

(*a*) £3·642183 (*c*) 3·68419 ft. (*e*) 9·614829 miles
(*b*) £4·58729 (*d*) 2·71882 tons (*f*) 3·41263 cwts.

DECIMALISATION

Decimalisation of Money, Weights, and Measures

22. The advantages of a decimal construction for tables of weights and measures have been recognised for years by the leading countries of the world which have adopted either the metric system (see Chapter VIII) first introduced by the French nearly 130 years ago, or slight modifications of this system. It is to be regretted that the United Kingdom still retains its ancient tables, involving as they do both in schools and business an enormous waste of time which could be devoted to other work.

23. The convenience of working in decimal form is so generally recognised that even in our own country we are led to decimalise both weights and measures, work in decimals, and then retransfer the result into the standard notation. The methods adopted are given below, and should be thoroughly understood and practised by the student until the various operations can be performed at sight.

24. *Money.*—In money, shillings, pence, and farthings are expressed as decimals of £1.

Shillings. Since 1/- = £$\frac{1}{20}$, it can be written £·05. Similarly 3/- = £·15, and 17/- = £·85, etc. The rule therefore follows immediately:

Rule 1.—*Multiply the number of shillings by* 5 *and mark off two decimal places from the result.*

25. *Accurate Decimalisation of Amounts less than a Shilling.*—Amounts less than 1/- should first be reduced to farthings, when they can be decimalised as follows:

$\frac{1}{4}d. = \frac{1}{24}$ of $6d. = \frac{1}{24}$ of $£·025 = £·001\frac{1}{24}$
or 1 farthing $= £1\frac{1}{24}$ thousandths.

Therefore

$2\frac{1}{4}d. = 9$ farthings $= £·009\frac{9}{24}$
$5\frac{3}{4}d. = 23$,, $= £·023\frac{23}{24}$
$7\frac{1}{4}d. = 29$,, $= £·029\frac{29}{24} = £·030\frac{5}{24}$
$11\frac{3}{4}d. = 47$,, $= £·047\frac{47}{24} = £·048\frac{23}{24}$

Instead of working out the remaining part of the decimal from the fraction with its denominator as 24, reduce the latter as follows:

$£·009\frac{9}{24} = £·009\frac{2\frac{1}{4}}{6} = £·009\frac{2·25}{6} = £·009375$
$£·023\frac{23}{24} = £·023\frac{5\frac{3}{4}}{6} = £·023\frac{5·75}{6} = £·023958\dot{3}$
$£·030\frac{5}{24} = £·030\frac{1\frac{1}{4}}{6} = £·030\frac{1·25}{6} = £·030208\dot{3}$
$£·048\frac{23}{24} = £·048\frac{5\frac{3}{4}}{6} = £·048\frac{5·75}{6} = £·048958\dot{3}$

If the student has carefully followed the working in the above examples, he should be prepared for the following rule:

Rule 2.—*To decimalise amounts less than* 1/-: *Express the sum in farthings and write this number as for the third decimal place, increasing it by one if the sum is* 6*d. or over* 6*d. Convert the pence and farthings (diminished by* 6*d. for amounts over* 6*d.) mentally to the decimal of a penny and divide by* 6*, writing the quotient in the fourth decimal place and onwards.*

e.g. $2\frac{1}{2}d. = £·010\frac{2·5}{6} = £·01041\dot{6}$.
$9\frac{1}{4}d. = £·038\frac{3·25}{6} = £·038541\dot{6}$.

After a little practice the whole decimal should be written down in one step only, and by a combination of Rules 1 and 2 any amount can be decimalised.

e.g. Express £2.17.11¼ as decimal of £1.

$$
\begin{array}{rr}
£2.17.11\frac{1}{4} = & £2·85 \\
 & 46875 \\
\hline
 & £2·896875 \\
\hline\hline
\end{array}
$$

This could have been written down directly in one line.

26. *Approximate Decimalisation (correct to 3 places of decimals).*—For many purposes the result correct to the nearest farthing is sufficiently accurate. This would not necessitate working farther than the third decimal place.

Since 1 farthing = £·001$\frac{1}{24}$, the result correct to 3 places = £·001 ; 7 farthings = £·007$\frac{7}{24}$, the result correct to 3 places = £·007 ; 12 farthings = £·012$\frac{12}{24}$, the result is equally correct as £·012 or £·013 ; 36 farthings = £·036$\frac{36}{24}$, the result is equally correct as £·037 or £·038.

Thus between 12 and 36 it is necessary to add *one* to the third place figure in order to be correct to this place, while above 36 *two* must be added for the same reason.

The rule for decimalisation is as follows :

Rule 3.—*Decimalise the shillings in the usual way, reducing amounts less than a shilling to farthings and writing this number for the third decimal place ; add one if the number is over* 12, *and two if over* 36.

The actual decimalisation can be written down in one line as follows :

Express £3.16.5¾ *as the decimal of* £1.

(*a*) Multiply the pence by 4, add 3. Total, 23 farthings. Since the number is over 12, add 1. Total, 24 farthings. Put down 4 in the third place, carry 2 to the second place.

(*b*) Multiply the shillings by 5 and add 2. Total, 80 + 2. Set down 82.

(*c*) Prefix the decimal point and write the number of £'s in front. Final result, £3·824.

27. *To convert the decimal of* £1 *into* £ *s. d.*

Rule 4.—*Write down the number of complete fives in the first two decimal places and call them shillings.*

The number remaining in the second and third places call farthings—deducting one if the number is over 13, two if over 37.

$$\text{e.g. } £\cdot877 = £\cdot85 + £\cdot027$$
$$= 17/- + 26 \text{ farthings}$$
$$= 17/6\tfrac{1}{2}$$
$$£\cdot65473 = (\text{approx.})\ £\cdot655 = 13/1\tfrac{1}{4}$$

Examples IIIb

(1) Express the following as decimals of £1 by Rule 2 :

(*a*) $3\frac{1}{4}$	(*d*) $2\frac{1}{2}$	(*g*) $9\frac{1}{4}$	(*k*) $7\frac{1}{2}$	(*n*) $10\frac{1}{2}$
(*b*) $9\frac{1}{2}$	(*e*) $5\frac{3}{4}$	(*h*) $3\frac{3}{4}$	(*l*) $8\frac{1}{4}$	(*o*) 5
(*c*) $1\frac{1}{4}$	(*f*) $11\frac{1}{2}$	(*j*) $4\frac{1}{4}$	(*m*) $6\frac{1}{2}$	(*p*) $4\frac{3}{4}$

(2) Express the following as decimals of £1 by Rules 1 and 2 :

(*a*) £9.6.$2\frac{3}{4}$	(*d*) £7.13.$11\frac{3}{4}$	(*g*) 18.9	(*j*) £3.13.$2\frac{1}{4}$
(*b*) 18.$2\frac{1}{2}$	(*e*) £3.19.4	(*h*) £2.15.$8\frac{1}{4}$	(*k*) £7.16.9
(*c*) 12.$5\frac{1}{4}$	(*f*) £5.11.6	(*i*) £9.16.$7\frac{1}{4}$	(*l*) £2.4.$3\frac{1}{2}$

(3) Work the above examples (1) and (2) correct to 3 places by Rule 3 :

(4) Express the following in £ *s*. *d*. :

(*a*) £3·742	(*d*) £2·6483	(*g*) £2·77809	(*j*) £·0331
(*b*) £4·563	(*e*) £7·584219	(*h*) £3·531	(*k*) £1·0029
(*c*) £8·971	(*f*) £2·3684	(*i*) £·8326	(*l*) £1·1111

28. *Decimalisation of Weights.*—For all practical purposes, wherever weights including tons are given, it is found sufficiently accurate to write correct to the nearest 7 lbs. In such cases the total weight can be expressed as the decimal of a ton by the following method :

29. Since cwts. bear to tons the same relation that shillings bear to pounds, they can be decimalised in the same manner. Similarly a quarter ($= \frac{1}{4}$ of cwt.) is equivalent to 3*d*., while 7 lbs. ($= \frac{1}{4}$ of a qr.) is equivalent to $\frac{3}{4}d$.

Thus 8 tons 14 cwts. 2 qrs. 21 lbs. is equivalent in decimal form to £8.14.8¼, and can be decimalised either correctly or approximately by Rules 1 to 3: i.e. 8 tons 14 cwts. 2 qrs. 21 lbs. = 8·734375 tons (Rules 1 and 2), or = 8·734 tons (Rule 3).

It is not necessary actually to write down the equivalent money form, thus the above could have been set down as follows.

Mentally, 2 qrs. 21 lbs. = 11 seven lbs., equivalent to 33 farthings. Therefore the first three decimal places can be filled immediately, ·734. Again, 33 farthings equal 8¼*d*., therefore (by Rule 2) the remaining figures are supplied by $\frac{2\cdot25}{6} = 375$.

EXAMPLES IIIc

(1) Express the following as decimals of 1 ton by the exact method (Rules 1 and 2):

(*a*) 3 tons 4 cwts. 3 qrs. 21 lbs.
(*b*) 6 tons 15 cwts. 1 qr. 7 lbs.
(*c*) 17 cwts. 21 lbs.
(*d*) 4 tons 5 cwts. 2 qrs. 14 lbs.
(*e*) 13 tons 16 cwts. 3 qrs. 7 lbs.

(2) Write the above as decimals of 1 ton by the approximate method (Rule 3).

30. When there are a number of examples of the same kind to be worked, the construction of tables such as the following will facilitate the work considerably:

No.	(*a*) lbs. = cwts.	(*b*) farthings = £'s	(*c*) inches = yards
1	·0089285714	·0010416	·027
2	·0178571428	·002083	·05
3	·0267857142	·003125	·083
4	·0357142857	·004166	·1
5	·0446428571	·0052083	·138
6	·0535714285	·00625	·16
7	·0625	·0072916	·194
8	·0714285	·0083	·2
9	·0803571428	·009375	·25

In forming these tables it is only necessary to work out the decimal for the 1 lb., farthing, or inch, the other numbers being derived from this by multiplying by 2, 3, 4, etc.

The method of using the tables is shown in the following examples:

(1) *Express* 2 *qrs.* 25 *lbs. as the decimal of* 1 *cwt.* (*correct to* 5 *places*):

2 qrs. 25 lbs. = 81 lbs.	80 lbs.	=	·714286 cwts.
	1 lb.	=	·008928 ,,
	81 lbs.	=	·72321

(*a*) The 80 lbs. is obtained by moving the decimal point in the 8 lbs.

(*b*) The figures were copied down to 6 places in order to obtain a figure to be carried forward.

(2) *Express* $7\frac{3}{4}$*d. as decimal of* £1 (*correct to* 5 *places*):

$7\frac{3}{4}$*d.* = 31 farthings	30 farthings	=	£·03125
	1 farthing	=	·001041
			£·03229

(3) *Express* 1 *ft.* 11 *inches as decimal of* 1 *yard* (*correct to* 4 *places*):

1 ft. 11 in. = 23 in.	20 in.	=	·55555 yard
	3 in.	=	·08333 ,,
			·6389 ,,

Examples III*d*

(1) From the table in para. 30 express the following as decimals of 1 cwt. (correct to 4 places):

(*a*) 2 qrs. 14 lbs.	(*d*) 18 cwts. 16 lbs.
(*b*) 3 qrs. 7 lbs.	(*e*) 3 cwts. 1 qr. 7 lbs.
(*c*) 4 cwts. 1 qr. 9 lbs.	(*f*) 5 cwts. 73 lbs.

(2) From the table in para. 30 write to the correct decimal of a £1: Examples 2 (*a*) to (*l*), Examples III*b*.

(3) Express the following lengths as decimals of 1 yard correct to 4 places (use column (*c*) in the above tables):

(*a*) 2 ft. 7 in.	(*d*) 17 in.	(*g*) 9 yds. 18 in.
(*b*) 34 in.	(*e*) 1 yd. 1 ft. 3 in.	(*h*) 2 ft. 9 in.
(*c*) 1 ft. 11 in.	(*f*) 6 yds. 2 ft. 1 in.	(*i*) 1 ft. 8 in.

(4) Construct tables on the above principle suitable for the following:

(*a*) To express yards as decimal of a furlong.
(*b*) To express square yards as decimal of an acre.
(*c*) To express square inches as decimal of a square foot.

From these tables express the following in decimal form correct to 5 decimal places:

(*a*) (i) 5 fur. 162 yds.	(*b*) (i) 2 roods 14 sq. yds.
(ii) 3 fur. 81 yds.	(ii) 364 sq. yds.
(iii) 2 fur. 57 yds.	(iii) 2,684 sq. yds.
(iv) 175 yds.	(iv) 341 sq. yds.
(v) 4 chains 7 yds.	(v) 4,372 sq. yds.

(*c*) (i) 73 sq. in.
(ii) 105 sq. in.
(iii) 3 sq. ft. 17 sq. in.
(iv) 21 sq. in.
(v) 137 sq. in.

CHAPTER IV

PRACTICE

SIMPLE PRACTICE

32. THE cost of any number of articles at £1 each can be written down immediately, and from the result we can find the cost whenever the price per article is a multiple or simple fractional part of £1.

E.g. the cost of 365 articles @ £1 each = £365; therefore the cost of 365 articles @ £3 each = £1,095; the cost of 365 articles @ 10/- = $\frac{1}{2}$ the cost @ £1 each = £182 10*s.*; the cost of 365 articles @ 3/4 = $\frac{1}{6}$ the cost @ £1 each = £60 16*s.* 8*d.*, and so on.

Again, having found these costs, we can derive from them others for smaller prices.

E.g. the cost of 365 articles @ 1/3 each = $\frac{1}{8}$ the cost @ 10/- each = £22 16*s.* 3*d.*; the cost of 365 articles @ 5*d.* each = $\frac{1}{8}$ the cost @ 3/4 each, or $\frac{1}{3}$ the cost @ 1/3 each = £7 12*s.* 1*d.*

Similarly, by a combination of suitable parts we can build up the cost for any price whatever, e.g.:

Example 1.—*Find the cost of* 365 *articles* @ 16/5 *each.*

	£	s.	d.
Cost @ £1 each	£365	0	0
Cost @ 10/- = $\frac{1}{2}$ cost @ £1	182	10	0
,, 5/- = $\frac{1}{2}$,, 10/-	91	5	0
,, 1/- = $\frac{1}{5}$,, 5/-	18	5	0
,, 5*d.* = $\frac{1}{12}$,, 5/-	7	12	1
,, 16/5 =	£299	12	1

33. It will be noted that all the fractional parts have been chosen so that they have unity as their numerators. They can therefore be obtained by simple division, since they are contained an exact number of times in the larger quantities from which they are derived. For this reason they are often referred to as *Aliquot Parts.* Fractions such as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$, etc., are avoided as they would involve multiplication as well as division.

34. The facility with which calculations are made by this method depends upon the number and suitability of the aliquot parts chosen. As a rule, the fewer the number of parts, the more quickly will the work be performed, though to avoid working with awkward divisors we may find it convenient to increase the number of parts, if by doing so we can obtain simpler divisors.

E.g. the aliquot parts chosen for 6/8$\frac{1}{2}$ may be:

(*a*) (6/8 = $\frac{1}{3}$ of £1) + ($\frac{1}{2}$*d.* = $\frac{1}{160}$ of 6/8).
(*b*) (5/- = $\frac{1}{4}$ of £1) + (1/8 = $\frac{1}{3}$ of 5/-) + ($\frac{1}{2}$*d.* = $\frac{1}{40}$ of 1/8).
(*c*) (4/- = $\frac{1}{5}$ of £1) + (2/6 = $\frac{1}{8}$ of £1) + (2$\frac{1}{2}$*d.* = $\frac{1}{12}$ of 2/6).

Of these both (*b*) and (*c*) will be found to be more convenient than (*a*), even though they include an extra step.

35. To enable the student to choose the most suitable aliquot parts he should practise dividing £1 by all its factors, then each of the aliquot parts so formed by its factors and so on, until he becomes quite familiar with all amounts which are likely to be of use in working.

36. Sometimes the work of calculation may be shortened by finding the cost at a higher price than the actual one given, and then subtracting

an aliquot part from the result. E.g. the aliquot parts for £3.18.4 may be either :

£3 + 10/- + 5/- + 3/4 or £4 − 1/8.

Example 2.—*Find the cost of* 2,756 *articles* @ £3 18*s*. 4*d*. *each.*

Cost @ £1 per article =	£2,756	0	0
			4
,, £4 ,, ,, =	11,024	0	0
1/8 = $\frac{1}{12}$ of £1	229	13	4
Cost @ £3 18*s*. 4*d*. =	10,794	6	8

37. To avoid working with unwieldy fractions in the pence column, it is often more convenient to work with these fractions in decimal form, or, better still, to adopt decimals for the whole of the shillings and pence.

For purposes of comparison the following example is worked in the three forms side by side.

Example 3.—*Find the cost of* $78\frac{7}{8}$ *oz. of silver plate* @ £2 6*s*. $7\frac{3}{4}$*d*. *per oz.* (*Answer correct to the nearest penny.*)

	(*a*)			(*b*)			(*c*)
	£	*s*.	*d*.	£	*s*.	*d*.	£
Cost @ £1 per oz. .	78	17	6	78	17	6	78·875
			2			2	2
Cost at £2 per oz.	157	15	0	157	15	0	157·750
,, 5/- = $\frac{1}{4}$ of £1 ,,	19	14	$4\frac{1}{2}$	19	14	4·5	19·71875
,, 1/- = $\frac{1}{5}$ of 5/- ,,	3	18	$10\frac{1}{2}$	3	18	10·5	3·94375
,, $7\frac{1}{2}$*d*. = $\frac{1}{8}$ of 5/- ,,	2	9	$3\frac{9}{16}$	2	9	3·56	2·46484
,, $\frac{1}{4}$*d*. = $\frac{1}{30}$ of $7\frac{1}{2}$*d*. ,,		1	$7\frac{23}{32}$		1	7·72	·08216
	183	19	2	183	19	2·28	183·9595

The answer correct to the nearest penny = £183 19*s*. 2*d*.

The methods (*b*) and (*c*) are both preferable to (*a*), and (*c*) is preferable to (*b*).

In (*b*) there is no need to work to more than two places of decimals, while in (*c*) we must work to five places to make sure that the third figure is correct.

EXAMPLES IV*a*

(1) Find the cost of:

(*a*) 28 tons of coal @ £1 17*s*. 6*d*. per ton.
(*b*) 36 ,, ,, ,, @ £1 18*s*. 4*d*. ,, ,,
(*c*) 17 ,, ,, ,, @ £2 2*s*. 6*d*. ,, ,,
(*d*) 39 ,, ,, ,, @ £1 13*s*. 4*d*. ,, ,,
(*e*) 42 ,, ,, ,, @ £1 16*s*. 8*d*. ,, ,,
(*f*) 108 ,, ,, ,, @ £2 1*s*. 4*d*. ,, ,,

(2) Find the cost of:

(*a*) 17½ cwts. of sugar @ £2 13*s*. 3*d*. per cwt.
(*b*) 13¾ ,, ,, ,, @ £2 7*s*. 5*d*. ,, ,,
(*c*) 57½ ,, ,, ,, @ £2 16*s*. 3*d*. ,, ,,

(3) What is the cost of:

(*a*) 387 sacks of potatoes @ 14/5 per sack.
(*b*) 295 ,, ,, ,, @ 13/9½ ,, ,,
(*c*) 1,165 ,, ,, ,, @ 10/8½ ,, ,,

(4) Find the price of:

(*a*) 78 pairs of boots @ 27/11 per pair.
(*b*) 3,000 books @ 2/8¼ each.
(*c*) 375 pairs of socks @ 2/10½ per pair.

(5) What must be paid for:

(*a*) 415 shares @ £2 6*s*. 11*d*. per share.
(*b*) 378 ,, @ £3 8*s*. 9*d*. ,, ,,
(*c*) 1,016 ,, @ £5 2*s*. 3*d*. ,, ,,

(6) Calculate the following:

(*a*) 37 days' pay @ 18/11¼ per day.
(*b*) 163 days' pay @ 17/3¼ per day.
(*c*) 57 hours' pay @ 1/3¾ per hour.
(*d*) 119 days' pay @ £1 1*s*. 3½*d*. per day.
(*e*) 37 hours' pay @ 3/6¼ per hour.

(7) Find the cost of:

(*a*) $37\frac{1}{2}$ square yds. of lino @ 4/$11\frac{1}{4}$ per sq. yd.
(*b*) $23\frac{2}{3}$,, ,, ,, ,, @ 7/$8\frac{3}{4}$,, ,, ,,

(8) Use the decimal method of Example 3(*c*) to work the following:

(*a*)	$367\frac{7}{8}$	articles	@ £12 3*s*. $5\frac{1}{4}$*d*.	each.
(*b*)	$2{,}931\frac{3}{4}$	,,	@ £3 17*s*. $11\frac{1}{4}$*d*.	,,
(*c*)	$2{,}834\frac{1}{3}$	,,	@ £6 13*s*. $5\frac{1}{2}$*d*.	,,
(*d*)	$4{,}293\frac{1}{4}$	,,	@ £3 12*s*. $10\frac{1}{4}$*d*.	,,
(*e*)	$284\frac{5}{6}$	,,	@ £6 18*s*. $9\frac{1}{4}$*d*.	,,
(*f*)	$315\frac{7}{12}$	,,	@ £2 11*s*. $8\frac{3}{4}$*d*.	,,

COMPOUND PRACTICE

38. Compound Practice is a method of finding costs of certain quantities of material when the price is quoted at so much per unit. In this case, the aliquot parts are based, not upon the price as in simple practice, but upon the unit for which the price is quoted.

Example 4.—Find the cost of 3 *miles* 5 *fur.* 160 *yds. of cable at* £83 15*s. per mile.* (*Answer correct to nearest penny.*)

	£	*s.*	*d.*	£
Cost of 1 mile =	83	15	0	83·75
			3	3
Cost of 3 miles .	251	5	0	251·25
do. 4 fur. = $\frac{1}{2}$ mile	41	17	6	41·875
do. 1 fur. = $\frac{1}{4}$ of 4 fur.	10	9	4·5	10·46875
do. 160 yds. = $\frac{1}{11}$ of 1 mile	7	12	3·27	7·61363
	£311	4	1·77	311·207

Cost correct to nearest penny = £311 4*s*. 2*d*.

Example 5.—*Find the cost of carriage on* 3 *tons* 17 *cwts.* 3 *qrs.* 21 *lbs.* @ £1 3*s.* 6*d. per ton.* (*Work correct to the nearest penny.*)

	£	s.	d.	£
Cost of 1 ton =	1	3	6	1·175
			3	3
Cost of 3 tons	3	10	6	3·525
do. 10 cwts. = $\frac{1}{2}$ of 1 ton		11	9	·5875
do. 5 cwts. = $\frac{1}{2}$ of 10 cwts.		5	10·5	·29375
do. 2 cwts. 2 qrs. = $\frac{1}{2}$ of 5 cwts.		2	11·25	·14687
do. 1 qr. = $\frac{1}{10}$ of 2 cwts. 2 qrs.			3·53	·01468
do. 14 lbs. = $\frac{1}{2}$ of 1 qr.			1·76	·00734
do. 7 lbs. = $\frac{1}{2}$ of 14 lbs.			·88	·00367
	4	11	6·92	4·579

Correct to nearest penny = £4 11*s.* 7*d.*

39. The above work could have been shortened by expressing the weight as the decimal of a ton, and working by simple practice as follows :

3 tons 17 cwts. 3 qrs. 21 lbs. = £3·896875.

Cost @ £1 per ton	£3·896875
do. 2/6 = $\frac{1}{8}$ of £1	·48711
do. 1/- = $\frac{1}{20}$ of £1	·19484
	£4·579 = £4 11*s.* 7*d.*

Note.—In examples such as the above, though the working is carried to five places, the last two figures are required only as a correction to the third place figure, and need only be added mentally to obtain the number to be carried forward.

40. The method of practice can also be extended to include examples such as the following :

Example 6.—*Find the weight of* 3 *miles* 4 *fur.*

120 *yds. of telegraph cable, if the weight per mile is* 12 *tons* 13 *cwts.*

	tons.	cwts.	qrs.	lbs.	tons.
Weight of 1 mile =	12	13			12·65
		3			3
	37	19			37·95
do. 4 fur. = $\frac{1}{2}$ of 1 mile	6	6	2		6·325
do. 110 yds. = $\frac{1}{8}$ of 4 fur.		15	3	7	·79062
do. 10 yds. = $\frac{1}{11}$ of 110yds.		1	1	21	·07187
	45	2	3		45·137

Note.—The decimal form of the ton can be retransferred mentally by reference to its money equivalent—see para. 29.

·137 tons = 2 cwts. + equivalent of 36 farthings.
= 2 cwts. + 12 seven lbs. = 12 cwts. 3 qrs.

Examples IV*b*

Find the value of each of the following;

(1) 3 cwts. 2 qrs. 21 lbs. @ £3 6*s*. 8*d*. per cwt.

(2) 4 tons 7 cwts. 2 qrs. 21 lbs. @ £8 5*s*. 0*d*. per ton.

(3) 3 qrs. 12 lbs. 8 oz. @ £3 16*s*. 8*d*. per qr.

(4) 33 miles 3 fur. 110 yds. @ £48 6*s*. 8*d*. per mile.

(5) 12 acres 3 roods 17 pls. @ £20 13*s*. 4*d*. per acre.

(6) 6 miles 3 fur. 120 yds. @ £29 18*s*. 0*d*. per mile.

(7) 4 miles 4 fur. 165 yds. @ £37 16*s*. 8*d*. per mile:

(8) 27 tons 3 cwts. 1 qr. 7 lbs. @ £8 6*s*. 8*d*. per ton.

(9) 9 cwts. 3 qrs. 14 lbs. @ £43 10*s*. 0*d*. per ton.

(10) 3 qrs. 3 bush. 2 pecks @ 178/- per qr.
(11) 5 qrs. 2 bush. 1 peck @ 208/- per qr.
(12) 12 yds. 1 ft. 11 ins. @ 27/6 per yd.
(13) 5 yds. 2 ft. 7 ins. @ 33/9 per yd.
(14) 3 sq. ft. 111 sq. ins. @ 47/9 per sq. ft.
(15) 19 galls. 3 qts. 1½ pts. @ 3/4 per gall.
(16) 2 years 146 days @ £350 per annum.

(17) If the yield per acre is 10 cwts. 2 qrs. 14 lbs., find the yield for 7 acres 3 roods 30 poles.

(18) Find the weight of 6 miles 3 fur. 130 yds. of cable @ 7 tons 12 cwts. per mile.

(19) Find the weight of 3 yds. 1 ft. 9 in. of piping @ 17 lbs. 8 oz. per ft.

(20) Find the dividend on £763 18*s*. at 13/9½ in the £.

(21) Find the dividend on £6,489 13*s*. @ 12/7½ in the £.

(22) Find the dividend on £137 15*s*. 8*d*. @ 6/5¾ in the £.

(23) Find the dividend on £94 18*s*. 7*d*. @ 3/4½ in the £.

CHAPTER V

INVOICES—FORMING A COMPANY FOR TRADING

41. Let us imagine a class formed into a Company; the rest of the class will constitute the firms with which the Company trades. It is better to form a Company than a partnership, as the number of partners a firm may have is limited by law; and it is safer, too, to form a limited company, for should our firm become bankrupt, the members (shareholders) of the Company will lose only the money they put into the business, i.e. their liability is limited. Should partners become bankrupt, however, even their private property may be taken to help to pay their debts.

After we have arranged for a Secretary, and elected three or four of our number as the Board of Directors, these officials must decide how much capital will be necessary for our business. Suppose they decide upon £1,000 divided into 1,000 shares of £1 each; they must now invite applications from intending shareholders for the number of shares they will individually require. These applications should in each case be accompanied by a cheque to cover 2/6 for each share required. After all applications are in, the directors will meet to allot the shares. Should there have been more shares applied for than 1,000, they will be

divided as far as possible proportionally, i.e. should 1,500 shares have been asked for, each applicant will get $\frac{1000}{1500} = \frac{2}{3}$ of the number for which he applied. On each applicant being notified of the number of shares allotted to him, he will forward a further cheque, making his payment up to 7/6 for each share he is to have. He must also be prepared to pay the further 12/6 per share when and how the directors may decide.

Now our Company is launched, and must procure a stock of the goods it wishes to sell. The shareholders, who must also be the clerks, should send orders to the firms from which they wish to buy. These firms must send invoices for the goods they supply drawn up in proper form. Invoices are important documents, by which the goods when they arrive are checked, and from which copies are made for the Company's books.

42. SPECIMEN INVOICE

116, WALLBROOK,
LONDON, E.C.
May 2, 1919.

THE COMMERCIAL CLASS CO., LTD.

Bought of A. Pupil

		£	*s.*	*d.*
54	yds. Black Serge @ 2/10 per yard	7	13	
45	,, Blue Serge @ 3/4 per yard	7	10	
72	,, Calico @ 1/7 per yard	5	14	
36	,, Silk @ 3/8 per yard	6	12	
		£27	9	

Per G.W.R.

The students can procure price lists from which ideas for orders may be gained.

43. Now the remainder of the class may send their orders to the Company, and practice in drawing up invoices will be afforded.

The Secretary of the Company should keep an account of the goods bought and sold, putting those bought on the left and those sold on the right, thus :

GOODS ACCOUNT

	Bought	£	*s.*	*d.*		*Sold*	£	*s.*	*d.*
May 1.	To Brown & Co.	17	10		May 8.	By Rowlands .	12	5	6
,, 10.	,, Phillips	3	15		,, 11.	,, Masters & Co. .	7	16	
,, 12.	,, Murray	8	8	6	,, 18.	,, Tasker	3	3	
,, 20.	,, Peters	9	10	6					

44. At the end of a month an imaginary stock may be taken, and inserted on the right-hand side. The two sides may then be totalled, and the amount by which the right-hand side exceeds the left will be the gross profits [1] of the Company for the month, and since this profit is made on a capital of £1,000, by dividing by 10 we can see how much that is on £100, i.e. the rate per cent.

Several exercises will be afforded by calculating the dividend to be paid to each shareholder.[2]

45. For quick calculations the following methods will be found useful in a large number of instances, and should be thoroughly known by the student.

To find the cost of a dozen articles.

Express the cost per article in pence—the cost

[1] See § 86 for distinction between gross and net profit.

[2] Numerous further exercises may be devised by the teacher—e.g. finding the percentage of profits on the goods sold ; balancing a Cash Book kept on similar lines to the Goods Account ; writing cheques. Practice also in filing and in business letter writing is also obtained.

per dozen will then be the same number of shillings.

Examples :

Cost per article	9*d.*	$8\frac{1}{4}$*d.*	$1/3\frac{1}{2}$ ($15\frac{1}{2}$*d.*)
Cost per dozen	9/-	8/3	15/6

This rule can be utilised in finding the cost for such numbers as 37 (3 dozen + 1); 99 (8 dozen + 3); Gross (12 dozen), etc., etc.

Example :

Find the cost of 59 lbs. of tea @ 2/5 per lb.
Cost = (29/- × 5) − 2/5 = 142/7 = £7 2*s.* 7*d.*

To find the cost of 20 *articles.*
Express the cost per article in shillings—the cost per score will then be the same number of £'s.

Examples :

Cost per article	4/-	3/9 ($3\frac{3}{4}$*s.*)	£1/4/$1\frac{1}{2}$ ($24\frac{1}{8}$*s.*)
Cost per score	£4	£3 15*s.*	£24 2*s.* 6*d.*

To find the cost of 240 *articles.*
Express the cost per article in pence: the cost per 240 articles will then be the same number of £'s.

Examples :

Cost per article	9*d.*	$1/3\frac{1}{2}$	$13/9\frac{1}{4}$
Cost per 240 articles	£9	£15 10*s.*	£165 5*s.*

This rule can be extended to finding the cost for such numbers as 120, 250, 360, 960, etc.

120 articles @ $1/11\frac{1}{2}$ = $\frac{1}{2}$ (£23 10*s.*) = £11 15*s.*
250 articles @ $1/6\frac{1}{2}$ = £18 10*s.* + 15/5 = £19 5*s.* 5*d.*
960 articles @ $2/4\frac{1}{2}$ = 4 (£28 10*s.*) = £114

Given the cost per day, to find the cost per year (365 days).

365 = 240 + 120 + 5.

∴ cost of 365 = cost of 240 + $\frac{1}{2}$ cost of 240 + cost of 5.

	£	*s.*	*d.*
365 days @ 1/10$\frac{1}{4}$ per day =	22	5	0
	11	2	6
		9	3$\frac{1}{4}$
	£33	16	9$\frac{1}{4}$

Given the cost per day, to find the cost per year, excluding Sundays (313 days).

313 = 240 + 60 + 12 + 1.

∴ cost of 313 = cost of 240 + $\frac{1}{4}$ cost of 240 + cost per doz. + cost of 1.

	£	*s.*	*d.*
313 days @ 8/8 per day =	104	0	0
	26	0	0
	5	4	0
		8	8
	£135	12	8

Frequent exercises must be worked on calculation of prices of goods from the price lists the student has obtained. Either mental calculation or the use of a minimum number of figures must be insisted upon. The common practice of showing every step of the working is frequently carried too far. In calculating the price of say 354 articles at 3/8 each, a few figures are necessary.

Thus:

354 @ 3/8 = £59 + 118/- i.e. $\left\{\begin{array}{l}\text{354 at 3/4, divide by 6} \\ \text{+ 354 @ 4}d\text{., divide} \\ \text{by 3}\end{array}\right.$

= £64 18 0

EXAMPLES Va

1. 48 articles	@ $4\frac{1}{2}d.$	$3\frac{1}{4}d.$	$6\frac{3}{4}d.$	each
2. 14 ,,	@ $5\frac{1}{4}d.$	$10\frac{3}{4}d.$	$1/1\frac{1}{2}$	,,
3. 63 ,,	@ $8\frac{1}{2}d.$	2/8	4/9	,,
4. 250 ,,	@ $2\frac{1}{4}d.$	$7\frac{1}{2}d.$	$8\frac{3}{4}d.$	,,

Invoices based on Price Lists.

5. Make out invoice for goods sold by Commercial Class Co. Ltd. to Messrs. F. Hall & Sons on June 8, 1919: 240 yds. shirting at 8/3 per dozen yards, 360 ditto at 10/9 per dozen yards, 120 yds. flannelette at 4/6 per dozen yds., 40 doz. reels assorted cottons at 3/3 per dozen reels.

6. Messrs. Masters & Sons of Halifax receive invoice from T. Harrage of London informing them that following goods have been dispatched per Midland Railway: 3 doz. tennis racquets at 21/6 each, 18 doz. balls @ 23/6 per doz., 28 golf clubs @ 5/9 each, 43 cricket bats @ 19/6 each, and 5 doz. cricket balls @ 4/3 each. Make out invoice.

7. Make out invoice on behalf of Messrs. Peters & Nelson, Bristol, to be sent to Messrs. Ritchie & Co. of London, telling them that the following goods have been dispatched per G.W.R.

54 quarter-lb. boxes Imperial Cigarettes @ 15/6 per lb.
38 lbs. Special Regal Mixture @ 12/- per lb.
1 gross Pouches @ 3/6 each.
6 doz. Briar Pipes @ 2/9 each.

8. The Unique Confectionery Co. purchase from Jas. Wall & Co. the following goods:

12 lb. boxes Assorted Creams @ 2/6 per lb.
36 lb. boxes Cream Caramels @ 2/8 per lb.
18 bottles Assorted Sugar Sweets @ 3/6 per bottle.
45 boxes Fancy Chocolates @ 6/6 per box.

Make out invoice.

9. Enumerate the uses of an invoice.

10. What is wrong with following invoice? Correct if necessary.

LONDON,
May 30, 1919.

J. PURCHASER

Bought of Messrs. Merchant & Sons

		£	s.	d.
38	copies Photographic Manual @ 1/6 per copy .	2	17	0
12	doz. Lightning Quarter Plates @ 1/4½ per doz .	0	14	6
18	doz. Lightning Half Plates @ 2/7½ per doz. .	2	6	3
60	packets "Glosso" Bromide Paper @ 9*d.* per pkt.	2	5	0
60	packets Magic Developer @ 4½*d.* per pkt . .	2	5	0
	£	10	8	9

THE CREDIT NOTE

46. It not infrequently happens in business that after goods have been received some portion of them have to be returned to the seller—perhaps because the wrong quality has been sent, or because those returned have been damaged in transit. The firm sending the goods has made a copy of the invoice in its Sales Book, and debited the account of the buyer with the amount of the invoice. These entries cannot be altered, since in book-keeping if corrections or alterations be necessary they are effected by making a compensating entry on the Credit side of an account if the mistake be on the debit side, and vice versa. Now the Credit Note is sent to the firm returning the goods to inform them that their account has been credited with the value of the goods returned, and that that amount should be deducted from the invoice on

making payment. Compare the following Credit Note with specimen invoice.

116, WALLBROOK,
LONDON, E.C.
May 10, 1919.

THE COMMERCIAL CLASS CO., LTD.

Credited by A. Pupil

		£	s.	d.
45	yds. Blue Serge @ 3/4 yd.—Wrong quality .	7	10	
12	yds. Silk @ 3/8 per yd.—Damaged . . .	2	4	
		9	14	

In order that a Credit Note might be readily distinguished from an invoice it is generally made out in red ink.

47. The commonest use of a Debit Note is to correct an undercharge, or the undercasting of an invoice, when such error is discovered after the invoice has been sent. Its form is similar to that of the credit note.

116, WALLBROOK,
LONDON, E.C.
May 10, 1919.

THE COMMERCIAL CLASS CO.

Dr. to A. Pupil

	£	s.	d.
To undercasting of Invoice date May 2 . .		10	

Such a document informs the firm receiving it that their account has been further debited with the amount specified, and they (the buyers) will therefore credit the account of the firm from which the goods were purchased.

EXAMPLES V*b*

1. Messrs. F. Hall & Co., when receiving the goods sent them by the Commercial Class Co. on June 10, 1919 (see Exercise on Invoices), find 20 yds. of shirting at 8/3 per doz. yds. have been damaged, and the whole of the cotton sent is of inferior quality. They return these goods, and receive a Credit Note from the Commercial Class Co. Make out the Credit Note.

2. The Commercial Class Co. find that the 360 yds. shirting sent was priced at 10/9 per dozen yds. instead of 11/9. Make out the debit note they will send to F. Hall & Sons.

Other exercises in making out Debit and Credit Notes may be provided by the student for himself from price lists as suggested for invoices.

ACCOUNT SALES

48. Many firms have agents in various towns or in foreign countries to sell goods on their behalf. These goods are not bought by the agent (or factor as he is called), but he is instructed to sell them for the consigning firm (the consignor) either at a specified price, or for the best price that he can get. After the goods are sold the factor deducts his commission (and any out-of-pocket expenses) before sending payment to consignor.

49. When the sale is complete the agent sends an Account Sales to the consignor, showing the prices realised by goods, all charges thereon, and the net proceeds, and how payment will be made.

The transaction is known as a "consignment," and in the consignor's ledger each consignment has a separate account in order that the profit on

each consignment may be readily seen, and such information as the agent who obtains the best prices, or whose charges are least, may be found.

The following is a specimen Account Sales (A/S).

ACCOUNT SALES OF THIRTY ROLLS OF TWEED CLOTH SOLD ON A/C OF MESSRS. SELLERS, LONDON

		Per yd.	£	s.	d.	£	s.	d.
480	yds. Grey Tweed . . .	3/9				90		
120	yds. Grey Tweed . . .	4/6				27		
240	yds. Brown Tweed . .	5/4				64		
240	yds. Fawn Tweed . .	4/8				56		
						237		
	Charges:							
	Insurance		5					
	Cartage		1	16				
	Commission @ 5 per cent.		11	17		18	13	
	Net Proceeds . . . Remitted per cheque					£218	7	

BRISTOL,
June 14, 1919.
P. HAZELTON & SONS.

EXAMPLES Vc

Note.—To calculate the 5 per cent. commission, take $\frac{1}{20}$th of gross cost of goods, i.e. 1/- in the £.

1. T. Howard & Co. sent on consignment to their agent H. Field of Newcastle the following goods: On June 10, 1919, they received an account sales, showing that 48 volumes Southey's *Life of Nelson* had been sold at 3/6 per volume, 84 complete editions of Dickens at 15/- per set, 120 complete Shakespeare at 10/6 and 100 complete Waverley Novels at 17/6 per set. The charges on the consignment were 5 per cent. commission for agent,

14/- for carriage, and 15/- for insurance. Make out A/S showing net proceeds.

2. Paul Fletcher & Sons sold on consignment for F. Harris & Co. the following. 240 pairs brogue shoes at 24/- per pair, 180 pairs marching boots @ 40/- per pair, 160 pairs leather leggings at 21/- per pair. The charges deducted were 5 per cent. commission, carriage £2 10*s*., storage £1, insurance 18/-. Make out A/S showing value of cheque P. Fletcher must send to F. Harris & Co.

3. H. Jarvis, a cycle agent, sold on behalf of the Cyclone Cycle Co. the following: 64 standard cycles @ £12 10*s*. each, 24 path-racers at £10 each, and 20 special cycles @ £15 each. Jarvis's commission was 5 per cent. on amount realised. Charges were: carriage £4, storage £3, insurance £2 10*s*. Make out A/S sent by Jarvis.

4. I receive a consignment of goods from T. Herd & Co., which I dispose of as follows: 36 lounge chairs at £5 10*s*. each, 12 occasional tables at £2 10*s*. each, 1 doz. overmantels @ £4 10*s*. each, various pictures £56.

My commission on the sale is 5 per cent., beside which I deduct the following charges insurance of goods £1 10*s*., carriage £4, storage £2 10*s*. Make out the A/S I send to T. Herd.

CHAPTER VI

AVERAGES

50. THE following list gives the value of the annual profits made by a certain firm for the years 1911–17:

1911 .	£80,166
1912 .	77,749
1913 .	76,301
1914 .	60,452
1915 .	62,797
1916 .	105,840
1917 .	108,518
	7)571,823
	£81,689

It will be seen from the list that the figures fluctuate from year to year, so that it is impossible to take any individual year as giving a correct indication of the firm's prosperity.

By adding together the whole of the profits for a period of years, however, and dividing the total by the number of years concerned, we obtain what is known as the *Mean* or *Average* profits for that period. Good and bad figures thus grouped together counteract each other, so that we avoid the danger of being misled by those of exceptional value.

In the above list the Average Profits for the seven years 1911–17 is £81,689.

51. The principal of averages is usually applied wherever records are kept of continually fluctuating figures, such as occur with regard to Temperature, Rainfall, Death Rate, Costs, Sales, Speeds, etc.

Example 1.—The accompanying table gives the Birth Rate and Death Rate for the years 1906–16 inclusive.

	Births.	Deaths.
	(per 1,000 population)	
1906 .	27·0	15·7
1907 .	26·3	15·4
1908 .	26·6	15·3
1909 .	25·7	15·0
1910 .	25·0	14·0
1911 .	24·4	14·8
1912 .	24·1	13·8
1913 .	24·1	14·3
1914 .	23·9	14·4
1915 .	22·2	15·6
1916 .	21·1	14·6

(*a*) *Find the Average Birth Rate for the* 5 *years* 1912–16.

Total Births per 1,000, 1912–16 = 115·4.

$\therefore$ Average per 1,000 = $\frac{115\cdot4}{5} = 23\cdot1$.

N.B.—In finding the average of numbers which vary only slightly—and have certain figures in common—we can save time by finding the average of the varying figures only, adding in the common figures at the end. This often enables the work to be performed mentally. Working the above example in this manner, we see that each number contains 20. Ignoring this and finding the average of the remainder, we get:

Average = $\frac{15\cdot4}{5} = 3\cdot1$

$\therefore$ Total Average = $20 + 3\cdot1 = 23\cdot1$.

(*b*) *Find the average Birth Rate for the* 5 *years* 1906–10.

Note each figure contains 25, adding the excess in each case:

Average = $25 + \frac{5\cdot6}{5} = 26\cdot1$.

Example 2.—*Six horses were sold so as to yield an average price of* £38 10*s. per horse. If the prices for the first five were* £50, £28 10*s.*, £33, £42, £35, *what was the selling price of the sixth horse?*

	£	s.	d.
Total price realised = £38 10*s.* × 6 =	231	0	0
Price for 5 horses = £50 + £28 10*s.* + £33 + £42 + £35 = . . .	188	10	0
$\therefore$ price of sixth horse = . .	£42	10	0

Examples VI

(1) The annual profits for a certain Company for the years 1914–18 were as follows: £115,223, £160,307, £124,846, £153,103, £157,195. Find the average annual profit for the period.

(2) In the example given in para. 50, find the average profit for the five years 1911–15, and compare with that for the five years 1913–17.

(3) Find the average annual profit made by a firm for the five years 1914–18, if the following were the individual annual profits: £7,112 8*s.* 3*d.*, £5,917 6*s.* 6*d.*, £12,253 7*s.* 6*d.*, £12,724 4*s.* 6*d.*, £16,306 7*s.* 1*d.*

(4) In the example (1), para. 51:

(*a*) Find the average births per 1,000 for the years 1907–16 inclusive.

(*b*) Find the average deaths per 1,000 for the years 1906–10 inclusive.

(*c*) Find the average deaths per 1,000 for the years 1912–16 inclusive.

(*d*) Find the average deaths per 1,000 for the years 1907–16 inclusive.

(5) In the example given in para. 19, column (*a*): Find the average imports for the five years 1913–17.

(6) Nine readings on a temperature card were as follows:

59·7°, 58·5°, 61·3°, 57·8°, 62·9°, 60·8°, 61·4°, 60·2°, 59·8°.

A tenth reading was obscured, but the average of the ten was given as 60·5°. What was the tenth reading?

(7) The daily earnings of a workman for 1 week (6 days) were: 19*s.* 8*d.*, 17*s.* 8*d.*, £1 0*s.* 6*d.*, 17*s.* 5*d.*, 18*s.* 3*d.*, £1 1*s.*

(*a*) What was his average daily wage?

(*b*) At this rate what would be his annual income (313 days)?

(*c*) If his wages for the first five days of another week were 18*s.* 4*d.*, 17*s.* 6*d.*, £1 0*s.* 5*d.*, 18*s.* 6*d.*, £1 1*s.* 6*d.*, what must he earn on the sixth day in order to keep to his average?

(8) The following prices were realised at a sale of horses: £108, £92, £86, two at £75 each, £140, six at 80 guineas each. What was the average price per animal?

Five more horses were then sold, bringing the average price up to £100. What was the average price of these horses?

(9) A firm kept the following figures relating to three of their motor delivery vans. Fill in the blank columns.

Capacity.	Annual Cost of Running.	Annual Mileage.	Av. Cost per mile.	Av. Cost per mile per ton.
	£ *s.* *d.*			
2 tons	227 10 0	12,000		
30 cwts.	178 5 0	10,000		
1 ton	159 15 0	10,000		

(10) The average price of wheat per quarter for the years 1910–17 inclusive was respectively: 31/8, 31/8, 34/9, 31/8, 34/11, 52/10, 58/5, 70/8. Find the excess of the average price for the last four years over the average price for the first four years.

(11) The average population of a seaside resort for the whole year was 11,900. For the months

November to March inclusive the town had its normal population. The remaining monthly averages from April to October were respectively, 11,400, 12,200, 12,800, 13,400, 14,400, 12,200, 11,400. What was the town's normal population?

(12) A person measured the length of a road in paces. Four different attempts gave 1,037, 1,044, 1,065, 1,058 paces. He then paced a distance of 100 yds., four attempts giving 112, 114, 110, 116 paces. By taking the average of both sets, calculate the length of the road in yards.

CHAPTER VII

CONTRACTED MULTIPLICATION AND DIVISION

52. Besides calculating costs by the method of Practice as shown in Chapter IV, we can, by decimalising both measures and money, use a method of direct multiplication. The ordinary processes of multiplying involve a large number of unnecessary figures, which are, however, avoided by the use of the contracted form of working shown below, e.g.:

Example 1.—*Find the cost of carriage on* 14 *tons* 6 *cwts.* 3 *qrs.* 7 *lbs.* @ £1 2*s.* 9*d.* per ton.

14 tons 6 cwts. 3 qrs. 7 lbs. = 14·340625 tons.
£1 2*s.* 9*d.* = £1·1375.

(a) *Ordinary Method*

14·3406	25
1·1375	
14·3406	25
1·4340	625
·4302	1875
·1003	84375
·0071	703125
£16·3124	609375

= £16 6*s.* 3*d.*

(b) *Contracted Method*

14·3406	25
1·1375	
14·3406	
1·4341	
·4302	
·1004	
·0072	
£16·3125	

£16 6*s.* 3*d.*

In working the above it must be remembered that the cost of carriage will be sufficiently accurate

if quoted correct to the nearest penny—which accuracy will be obtained if the decimal in the product is correct to the third place.

In (*a*), therefore, we need only have worked as far as the fourth decimal figures, and could have dispensed with those to the right of the vertical line—provided we made a correction on account of their omission. The method by which the work is performed is shown in the contracted form (*b*).

53. *Contracted Multiplication.*

Since no figures are required in the working beyond the fourth decimal place, each digit of the multiplier should begin multiplying the top line, so that the right-hand figure of its product rests in the fourth place, e.g.:

The 1 unit begins to multiply at the 6 (4th place fig. × units = 4th place fig.).

The 1 tenth begins to multiply at the 0 (3rd place fig. × 1st place fig. = 4th place fig.).

The 3 begins to multiply at the 4 (2nd place fig. × 2nd place fig. = 4th place fig.).

The 7 begins to multiply at the 3 (1st place fig. × 3rd place fig. = 4th place fig.).

The 5 begins to multiply at the 4 (units place fig. × 4th place fig. = 4th place fig.).

From this it is seen that as each successive figure of the multiplier moves one place from left to right, so the figure at which to commence multiplying on the top line moves one place from right to left. Therefore having fixed the first starting-point, the others can be obtained by ticking off each figure, as used.

54. To ensure the answer being correct to the third decimal place, the fourth place figure should

itself be corrected on account of the fifth place figure. This can readily be done by multiplying the next right-hand figure, and simply bringing forward the required digit from the product, e.g.:

In the 4th line of the working, the 7 commences multiplying at the 3, but a correction is made on account of the right-hand figure 4.

Mentally: $7 \times 4 = 28$. Bring forward a 3. (This is more correct than 2.)

55. *De Morgan's Method.*

This is the same as the above, save that the multiplier is reversed, and written with its decimal point omitted underneath the multiplicand, so that its units figure is in the last decimal place required in the working.

The above example worked in this manner would be as shown:

```
14·340625
  57311
 143406
  14341
   4302
   1004
     72
£16·3125
```

(*a*) Since the working is to be carried to four decimal figures, the units figure of the 1·1375 is written underneath the fourth place figure of the top line, and commences to multiply at this figure. Similarly, by reversing the remaining figures of the multiplier they will now be under the respective figures at which they should commence to multiply. The reason for this is obvious if we compare the working with that of para. 52.

(*b*) All products must commence from the 4th place.

(*c*) The decimal point is not required until the end, when the necessary position can be counted off.

Example 2.—*Find the cost of* 306·857 *tons* @ £29·384 *per ton.* (*Correct to the nearest penny.*)

```
   306·857
      4839|2
 6137 1400
 2761 7130
   92 0571
   24 5486
    1 2274
 9,016·6861
= £9016 13s. 9d.
```

Note.—

(*a*) Working to four places, the 9 units is placed under the 4th place.

(*b*) The 2 commences to multiply at the figure above it, so that there are two ciphers before it multiplies the 7.

Example 3.—*Find the cost of* 2963·88 *cwts.* @ £·487 *per cwt.* (*Correct to the nearest penny.*)

```
  2,963·88..
        784.
  1,185 5520
    237 1104
     20 7472
 £1,443·4096
= £1,443 8s. 2d.
```

(*a*) The units figure would have been placed under the 4th decimal place, so that the 4 is written in the 3rd place.

(*b*) Though there are no digits in either line actually in the 4th place, the results are still written from this position.

Examples VII*a*

Work the examples in Ex. IV*b* by the above method of contracted multiplication (correct to the nearest penny).

56. *Contracted Division.*—Consider the following example:

The annual output of coal in the British Isles for the year 1912 *was* 264,595,395 *metric tons. If the number of persons employed in production was* 1,072,393, *find the average output per person.* (*Correct to the nearest ton.*)

By the ordinary methods of division the working

is as shown under (*a*). This is not only long and cumbersome, but also contains many figures which have no bearing upon the answer. Thus, all figures to the right of the vertical line could be omitted without affecting the accuracy of the result. For this and similar examples, therefore, we are led to adopt a *contracted method* of division, such as is shown under (*b*), where all superfluous figures are omitted.

(*a*)

```
1·072393)264·5|95395(246·7
          2144|786
          ----|---
           501|1679
           428|9572
           ---|----
            72|21075
            64|34358
            --|-----
             7|867170
             7|506751
             -|------
              |360419
```

Correct to nearest ton = 247.

(*b*)

```
               246·8|
   1·072393)264·6|
               2145 |
               ---- |
                501 |
                429 |
                --- |
                 72 |
                 64 |
                 -- |
                  8 |
                  8 |
                  - |
                  · |
```

Correct to nearest ton = 247.

57. The method of procedure is as follows :

(*Generally*)	(*In the above Example*)
(1) Write the Divisor so that there is one figure to the left of the decimal point—and alter the Dividend accordingly.	The old Divisor 1,072,393 is re-written as 1·072393 by moving the decimal point 6 places to the left. The same process alters the Dividend from 264,595,395 to 264·595395.
(2) Find one more figure in the Quotient than is actually required in the answer.	The answer is required correct to the nearest ton, therefore the quotient is obtained to the first decimal place.
(3) Retain only as many figures to the right of the decimal point of the Dividend	Only the first decimal figure of the Dividend need be kept. This is corrected from 5 to 6

as are required to the right of the decimal point in the Quotient; if necessary, correct the last figure retained.

on account of the omitted figures.

(4) By a trial division determine the value of the first figure of the Quotient, and write it in its correct position above the Dividend. Now find the number of figures still required in the Quotient, and in the Divisor count off and retain the same number of figures after the decimal point —cancelling the remainder.

A trial division gives 2 *hundreds* as the first figure. Writing this above the *hundreds* figure of the dividend, it is seen that there are 3 more figures in the Quotient still to be found. Counting off this number after the decimal point of the Divisor, we retain the 1072 and cancel the 393.

(5) After each division cancel the right-hand figure of the Divisor—retaining it mentally, however, when multiplying to see if there is any figure to carry forward.

In the multiplication of 1072 by 2, we multiply the cancelled 3 mentally. Though this does not actually give a figure to carry forward, the 6 is greater than half value and 1 must be brought forward.

Example 4.—*In* 1911 *the value of the Total Imports into the United Kingdom was* £680,157,527. *How much did this average per head if the population was* 45,221,615? (*Work correct to the nearest penny.*)

```
              15·0405
4·5221615)68·0158
             45·2216
             22 7942
             22 6108
                1834
                1809
                  25
                  23
                   2
```

Correct to the third decimal place = £15·041

(*a*) Since the decimal point is moved seven places to the left in the Divisor, it must be similarly moved in the Dividend.

(*b*) The answer is required correct to three decimal figures, so that four figures should be obtained after the decimal point in the Quotient; therefore only four figures need be retained after the point in the Dividend.

∴ average per head, correct to nearest penny = £15 0*s*. 10*d*.	(*c*) The 7, being thus the last figure retained, is corrected to 8 on account of the 5 following.

EXAMPLES VII*b*

Supply the figures required in the following examples by working with the Contracted Method of Division.

(1) In the columns below, find, correct to the nearest ton, the average output of coal per person employed.

Country.	Output in Metric Tons. 1903	1912	No. of Persons Employed. 1903	1912	Av. Output per Person. 1903	1912
(*a*) British Isles	234,030,784	264,595,395	828,968	1,072,393		
(*b*) France .	34,906,418	41,145,178	167,213	202,365		
(*c*) German Empire .	162,457,253	255,810,100	522,823	718,673		
(*d*) U.S.A. .	324,191,615	484,864,901	566,260	722,622		

(2) From the figures below find, correct to the nearest penny, the average receipts per train mile.

	Total Goods Receipts	Total Goods Mileage.	Av. Receipts per Train Mile.
(*a*) 1903 .	£55·11 millions	161·63 millions	
(*b*) 1913 .	£66·64 ,,	161·68 ,,	

(3) The figures below give the value of the mineral output and the population of certain

countries in 1910. Find the value per head of population. (Correct to the nearest penny.)

Country.	Value.	Population.	Av. Value per Head.
(*a*) U.S.A. . .	£411 millions	91,972 thousands	
(*b*) U.K. . .	£140 ,,	45,370 ,,	
(*c*) British Empire	£253 ,,	435,000 ,,	
(*d*) Germany .	£129 ,,	64,925 ,,	

(4) Find to the nearest unit the average number of people per square mile.

Country.	Area in Sq. Miles.	Population in 1911.	Av. No. per Sq. Mile.
(*a*) India . .	1,802,577	315,086,372	
(*b*) England . .	50,866	34,038,537	
(*c*) Wales . .	7,474	2,031,955	
(*d*) Scotland . .	29,798	4,760,904	
(*e*) Ireland . .	32,586	4,390,219	

(5) Calculate the population of the United Kingdom by means of the following figures. (Work correct to nearest 100,000.)

	Exports of United Kingdom.		Population.
	Total Value.	Proportion per head.	
	£	£ *s.* *d.*	
(*a*) 1907 . .	426,035,083	9 14 10	
(*b*) 1908 . .	377,103,824	8 11 0	
(*c*) 1909 . .	378,180,347	8 9 11	
(*d*) 1910 . .	430,383,772	9 11 8	
(*e*) 1911 . .	454,119,298	10 0 7	

CHAPTER VIII

THE METRIC OR DECIMAL SYSTEM

58. In the Metric System of Weights, Measures, and Coinage, when we count from one we introduce a new unit or term on reaching ten, another on reaching one hundred, and so on. How much more easy would our calculations be if 10 inches made one foot, 10 feet one yard! This system, to which we are so accustomed in ordinary notation, and which probably originated in early times from the use of fingers in counting, has been adopted in many countries, thereby simplifying all arithmetical processes connected with weights, measures, and money.

59. The metric system is so called because all the units of its weights and measures are based upon the metre. Another advantage of the system is the use of prefixes common to all the tables. Thus in the long measure the unit is the metre, which is divided into tenths, hundredths, thousandths, called respectively the decimetre, centimetre, and millimetre, the prefixes used being derived from the Latin words meaning ten, hundred, and thousand. The measures greater than the metre are the Dekametre, the Hectometre, and the Kilometre (from the Greek words meaning 10, 100,

and 1,000), 10, 100, and 1,000 metres respectively. The table thus is .

10 millimetres	= 1 centimetre,	written	mm., cm.
10 centimetres	= 1 decimetre,	,,	dm.
10 decimetres	= 1 METRE,	,,	m.
10 metres	= 1 Dekametre,	,,	Dm.
10 Dekametres	= 1 Hectometre,	,,	Hm.
10 Hectometres	= 1 Kilometre,	,,	Km.

These prefixes are common to all weights and measures. The unit of weight is the gramme, which is the weight of a cubic centimetre of water at 4° Centigrade; a litre, the unit of capacity, equals a cubic decimetre.

EXAMPLES VIII*a*

1. Construct the table of weight.
2. Construct the table of capacity.
3. Reduce to metres, 5 Km. 4 Hm. 6 Dm. 3 m.; 4 Km. 3 Dm.
4. Reduce to millimetres, 70 m.; 4 m. 3 dm. 2 cm.
5. Reduce to centilitres, 4 Kl. 8 Hl. 9 Dl. 7 l. 3 dl. 4 cl.
6. Reduce to decigrammes, 3 Kg. 4 Dg. 9 g. 5 dg.
7. How many metres in 3,179 mm.?
8. How many kilometres in 4,179,547 mm.?
9. Reduce 73,965 decigrammes to kilogrammes.

60. The English equivalents of the units of the metric system are .

1 metre	=	39·37 ins.
1 gramme	=	·035 ozs.
1 litre	=	1·76 pints.

A better equivalent to remember than that of the gramme is—

$$1 \text{ Kg.} = 2{\cdot}2 \text{ lbs.}$$

Examples VIII*b*

1. Give the English equivalents of 1 dm., 1 cm., 1 mm. in inches.
2. Give the English equivalents of 1 Dm., 1 Hm., 1 Km. in yards.
3. Give the English equivalent of 1 dl. 1 cl. in pints.
4. Give the English equivalent of 1 Kl. in bushels.
5. Which is the greater, and by how many yards (to nearest yard), 1 Km. or 1 mile ?
6. Express 1 Km. as the fraction of a mile. To what fraction with one figure as numerator and denominator is this approximately equal ?
7. In expressing heavy weights a metric tonne (= 2,205 lbs.) is used. How much is this less than an English ton ? Express the metric tonne as an approximate simple fraction of the ton.
8. Express one English quarter (capacity) as the approximate fraction of a Kilolitre.
9. Express 5 cwt. 3 qrs. 14 lbs. in Kg. (to nearest unit).
10. How many metres are there in a mile ?
11. Which is the greater, and by how much (in pints), 8 Hl. 3 Dl. 7 l. or 23 bushels, 3 pecks, 1 gallon, 2 quarts ?
12. How many Kg. in 3 cwt. 3 qrs. 3 lbs ?

61. *Metric Measures of Area and Volume.*—We know that in measuring area of a square we multiply the number representing the length of the side by itself. Thus if the side of a square be a foot

in length, it contains one square foot or 12×12 square inches.

Similarly, in the metric system the number of square decimetres in a square metre is $10 \times 10 = 100$, and the table proceeds in hundreds, thus :

100 sq. mm. = 1 sq. cm.
100 sq. cm. = 1 sq. dm., etc.

The unit of square measure is the "are," which equals 100 sq. metres. The hectare (10,000 sq. metres) is also used. Areas are, however, generally quoted in terms of the square metre. The unit of *Cubic Measure* is the cubic metre (sometimes called the stere) = $10 \times 10 \times 10$ cubic decimetres.

Examples VIII*c*

1. Construct the tables of square and cubic measures.

2. Express in square metres :

(*a*) 4·365 square hectometres.
(*b*) 5,678 square decimetres.
(*c*) 0·056 square kilometres.
(*d*) 37,364 square centimetres.

3. One acre contains 4,840 sq. yds. How many sq. metres does it contain, to nearest unit ?

4. Express in cubic metres :

(*a*) 5·718 cubic hectometres.
(*b*) 35,650 cubic decimetres.
(*c*) 0·0465 cubic kilometres.
(*d*) 417,165 cubic centimetres.

5. A cubic centimetre of water weighs 1 gramme. What is weight of 1 cubic decimetre ? Thence show relation between 1 litre and 1 Kg.

6. How many kilograms of water are there in a vessel containing 5,370 cubic centimetres?

62. *Decimal Coinage.*—The coinage of France has for its unit the franc. The centime is $\frac{1}{100}$th part of a franc.

The value of the franc in English money is normally about $9\frac{1}{2}d.$ or £1 = 25 francs 22 centimes, written fr. 25·22.

Examples VIII*d*

1. Make out an invoice:

Adolphe et Cie have sold to M. Arnaud Pincer the following:

32·5 Kg. prunes	@ 1 fr. 20 per Kg.
20 Kg. chocolate	@ 4 fr. 25 per Kg.
45 litres wine	@ 4 fr. per litre.
72 Kg. coffee	@ 5 fr. 10 c. per, Kg.

2. What is the value of 765 fr. 25 c. in £ *s. d.* (£1 = 25 fr. 22 c.)?

3. What is value of 7,416 fr. in £ *s. d.*?

4. Find value of £79 18*s.* $10\frac{1}{2}d.$ at same rate in francs.

5. An English price list marks certain goods at 18*s.* The same article is marked in a French catalogue at 22 fr. Which is cheaper, and by how much to nearest penny (£1 = 25 fr.)?

6. A French firm offers goods at 215 frs., an English firm at £8 17*s.* 6*d.* If carriage of goods from France costs 5 frs., what should I gain by buying from France (£1 = 25 fr.)?

7. Taking 1 fr. = $9\frac{1}{2}d.$, 1 metre = $39\frac{1}{3}$ ins., 1 Kg. = $2\frac{1}{5}$ lbs., 1 litre = $1\frac{3}{4}$ pints, find (*a*) by

what factor should we multiply shillings per yard in order to convert the price to francs per metre.

Method : 1*s.* per yd. = 3″ for 1*d.*

$$\therefore 1 \text{ metre costs } \frac{39\frac{1}{3}}{3}d.$$

$$= \frac{39\frac{1}{3}}{3\times 9\frac{1}{2}} \text{ francs} = \frac{118}{3\times 3} \times \frac{2}{19}$$

$$= \frac{236}{171} = 1{\cdot}38.$$

(*b*) By what factor should we multiply francs per metre to obtain shillings per yard ?

(*c*) By what factor should we multiply francs per Kg. to convert to shillings to per lb. ?

(*d*) By what factor should we multiply pence per lb. to convert to francs per Kg. ?

(*e*) By what factor should we multiply francs per litre to convert to pence per pint ?

(*f*) By what factor should we multiply pence per pint to convert to francs per litre ?

8. By using above factors, give costs of following in francs per metre . 3/6, 2/8, 7/10, 12/5, 19/6, £1 4*s.* 7*d.* per yd.

9. Give costs of following in shillings and pence per yd. : 8 fr., 3 fr. 50 c., 1 fr. 20 c., 12 fr. 25 c., 10 fr. 10 c. per metre.

10. Give costs of following in shillings and pence per lb. : 2 fr. 70 c., 4 fr. 75 c., 9 fr. 20 c., 24 fr. 15 c. per Kg.

11. Give costs of following in fr. per Kg. : 8*d.*, 1/6, 5½*d.*, 3/2 per lb.

12. Give costs of following in francs per litre : 7*d.*, 3/4, 6½*d.*, 1/4 per pint.

13. Give costs of following in pence per pint : 9 fr., 2 fr. 50 c., 1 fr. 75 c., 19 fr. 40 c. per litre.

CHAPTER IX

RECTANGULAR AREAS AND VOLUMES

63. THE simplest form of surface from the point of view of measurement is the rectangle. This is the shape usual to the floor and walls of a room, the pages of a book, doors, windows, etc.

By definition, a rectangle is stated to be a four-sided figure with all its angles right angles. From this it can easily be shown that its opposite sides are equal. Its area, *i.e.* the number of square inches, square feet, etc., which it contains, is found by multiplying the number of linear units in its length by the number of linear units in its breadth, or, as it is usually written :

$$\textit{Area of a Rectangle} = \textit{Length} \times \textit{Breadth}.$$

64. When making calculations, care should be taken that both length and breadth are expressed in terms of the same linear unit ; thus, both should be in feet, or in inches, or yards, and so on. The resulting area will then be expressed in terms of the square unit, e.g.: A rectangle 9 ins. long by 8 ins. wide has an area of $(9 \times 8) = 72$ sq. ins. The same dimensions expressed in feet would give area $= (\frac{3}{4} \times \frac{2}{3}) = \frac{1}{2}$ sq. ft. These results are the same since 72 sq. ins. $= \frac{1}{2}$ sq. ft.

65. If the area of a rectangle and the length of one side are known, the length of the other side can also be determined, thus :

$$\text{Length} = \frac{\text{Area}}{\text{Breadth}} \text{ or Breadth} = \frac{\text{Area}}{\text{Length}}.$$

Example 1.—*Find the area of a roll of wall-paper 12 yds. long by 27 ins. wide.*

$$\text{Area} = (12 \times \tfrac{3}{4}) = 9 \text{ sq. yds.}$$

Example 2.—*Find the length of a piece of carpet if its area is 250 sq. ft. and its width is 30 ins.*

$$\text{Length} = \frac{250}{2\frac{1}{2}} = 100 \text{ ft.}$$

Example 3.—*Find the cost of staining a border 18 ins. wide round a rectangular room of length 20 ft., breadth 15 ft. Cost of staining, 6d. per sq. yd.*

Length of the unstained part = 20 ft. less twice 18 ins. = 17 ft.

Breadth of the unstained part = 15 ft. less twice 18 ins. = 12 ft.

Area of whole room = (20 × 15) = 300 sq. ft.

Area of unstained portion = (17 × 12) = 204 sq. ft.

∴ Area stained = (300 − 204) = 96 sq. ft.

$$\therefore \text{ Cost} = \frac{96 \times 6}{9} \text{ pence} = 5s.\ 4d.$$

66. In the laying of a floor, joiner's work is measured in terms of the *square* of 100 superficial feet. The number of floor boards required depends both upon the dimensions of the boards and the manner in which they are laid, e.g. a square of flooring may require either $12\frac{1}{4}$, $12\frac{1}{2}$, 13, $13\frac{1}{2}$, or 14 boards if laid with 12 ft. deals, while 17 or 18 are necessary if 12 ft. battens are used.

Example 1.—*How many boards are required to cover a floor $20\frac{1}{2}$ ft. long by $15\frac{3}{4}$ ft. wide, calculating $12\frac{1}{2}$ boards per "square"?*

Area of room $= \left(\frac{41}{2} \times \frac{63}{4}\right)$ sq. ft.

$\therefore$ No. of squares $= \dfrac{41 \times 63}{8 \times 100}$

and no. of boards $= \dfrac{41 \times 63 \times 12\frac{1}{2}}{8 \times 100}$

The above is set out in three lines to show the steps in the reasoning. In working it could have been set down immediately in one line, thus:

$$\text{No. of boards} = \frac{41}{2} \times \frac{63}{4} \times \frac{\overset{1}{\cancel{12\frac{1}{2}}}}{\underset{8}{\cancel{100}}} = 41 \times \frac{63}{64} = 41.$$

N.B.—The correct arithmetical answer gives $40\frac{23}{64}$ boards, but as 41 are actually required, the fractional portion is counted as an extra board. Unless otherwise asked for, all problems similar to the above in this book should be dealt with in the same manner.

Example 2.—*Find the cost of panelling a ceiling* 46 *ft.* 8 *ins. long by* 31 *ft.* 8 *ins. wide at* 3*s.* 6*d. per panel, each panel being* 28 *ins. long and* 20 *ins. wide.*

Area of ceiling $= (46\frac{2}{3} \times 31\frac{2}{3})$ sq. ft.

Area of each panel $= (2\frac{1}{3} \times 1\frac{2}{3})$ sq. ft.

No. of panels reqd. $= \dfrac{46\frac{2}{3} \times 31\frac{2}{3}}{2\frac{1}{3} \times 1\frac{2}{3}} = \dfrac{140 \times 95}{7 \times 5} = 380$

$\therefore$ Cost $= 3s.\ 6d. \times 380 = £66\ 10s.$

Examples IX*a*

(1) Find the area of the following fields in acres and sq. yds. if their lengths and breadths are respectively:

(*a*) 190 yds. 121 yds. (*c*) 214 yds. 135 yds.
(*b*) 150 yds. 84 yds. (*d*) 98 yds. 57 yds.

(2) How many floor boards are required for the following rooms ?—

(*a*) Length 32 ft., width 16 ft. 4 ins., allowing 13 planks per 100 sq. ft.

(*b*) Length 15 ft., width 12 ft. 6 ins., allowing 13 planks per 100 sq. ft.

(*c*) Length 18 ft., width 14 ft. 9 ins., allowing $12\frac{1}{2}$ planks per 100 sq. ft.

(*d*) Length 22 ft. 8 ins., width 4 ft. 6 ins., allowing 14 planks per 100 sq. ft.

(*e*) Length 12 ft. 9 ins., width 9 ft. 6 ins., allowing 18 planks per 100 sq. ft.

(3) The roof of a greenhouse is 50 ft. long by 10 ft. 6 ins. deep. Find the cost of glazing it, if the glass is 57*s.* 6*d.* per 100 sq. ft. (allow $\frac{1}{8}$th for overlapping). (Answer correct to the nearest shilling.)

(4) Pavements 5 ft. 9 ins. wide are to be laid along both sides of a road 128 yds. long. Find the cost of paving at 2*s.* 3*d.* per sq. yd.

(5) A vestibule 8 ft. 6 ins. long by 6 ft. 9 ins. wide is laid with tiles each 4 ins. by 3 ins. Find the total cost at 1*s.* 10*d.* per dozen tiles.

(6) Three ornamental gardens each 50 ft. long by 40 ft. wide are contained in a courtyard 80 yds. long by 30 yds. wide. How many paving stones 2 ft. 6 ins. long by 20 ins. wide are required to cover the remainder of the yard ?

(7) A field 150 yds. long by 95 yds. wide is to be cut up into 60 equal allotments. What rent should be charged for each at the rate of £5 10*s.* per acre (correct to nearest penny) ?

(8) What is the largest number of cards 6 ins. long by $3\frac{1}{2}$ ins. wide which can be cut from a sheet 40 ins. long by 25 ins. wide ? How many sq. ins. *waste* will this leave ?

(9) The maximum dimensions permissible for a

football field are 130 yds. by 100 yds., the minimum 100 yds. by 50 yds. Find the difference in price of returfing one of maximum and one of minimum size, if turfs measure 18 ins. by 10 ins. and cost 11*s*. 3*d*. per 100.

(10) A garden 150 ft. long by 80 ft. wide contains a central flower-bed 30 ft. long by 20 ft. wide, around which is a path 3 ft. wide. Find the area of (*a*) the central flower-bed, (*b*) the path, (*c*) the remainder of garden.

(11) In an athletic ground the following pitches were returfed at a cost of £800 : two, each 120 yds. long by 80 yds. wide ; one, 100 yds. by 80 yds. ; and one 100 yds. by 60 yds. Find correct to the nearest penny the cost of turfing per sq. yd.

(12) Find the number of tiles each 15 cm. by 10 cm. required to pave the floor of a vestibule 5·5 metres long by 2·3 metres wide.

(13) Find the cost of painting a wall 12·4 metres long and 2·3 metres high at 4 frs. 75 centimes per sq. metre.

Express the result in £ *s*. *d*., given that £1 = 29·3 francs.

(14) Find the cost of returfing a lawn 20 metres long by 10·5 metres wide with turfs each 45 cms. by 30 cms., given the cost of turfs is 24 fr. 75 c. per 100.

(15) The price of a building plot 30 yds. long by 25 yds. wide is £874. Express this rate in terms of francs per hectare, given that £1 = 28·7 francs.

67. *Area of the Walls of a Room.*—In finding the area of the walls of a rectangular-shaped room, we can, instead of taking the area of each of the four walls separately, imagine them to be unfolded

and laid out flat so as to form one long rectangle, the length of which is equal to the perimeter of the room, i.e. the total distance round it. We can, therefore, write as follows:

Area of the four walls = perimeter of room × height of room.

Since opposite walls have the same length—

$$\text{perimeter} = 2 \times (\text{length} + \text{breadth}).$$

Example 1.—*Find the area of the walls of a room of which the dimensions are: length* 18 *ft.* 6 *in., width* 15 *ft.* 6 *in., height* 10 *ft.*

$$\begin{aligned} \text{Perimeter} &= 2(18\tfrac{1}{2} + 15\tfrac{1}{2}) = 68 \text{ ft.} \\ \text{Height} &= 10 \text{ ft.} \\ \therefore \text{ Area} &= (68 \times 10) = 680 \text{ sq. ft.} \end{aligned}$$

68. *Papering the Walls of a Room.*—In estimating the amount of paper required to cover the walls of a room, allowance must be made for the space occupied by doors, windows, fireplace, etc. Wall-papers are sold in rolls 12 yds. long and generally 21 ins. wide, giving an area of 7 sq. yds. per roll; so that, knowing the number of sq. yds. to be papered we can obtain the number of rolls required by dividing by 7. Matching the pattern in adjacent strips causes a certain amount of waste, for which it is usual to allow one roll for about every ten used.

Example 2.—*If the room in Example* 1 *contains a door* 7 *ft. by* 4 *ft., a window* 8 *ft. by* 6 *ft., and a fireplace* 6 *ft. by* 4 *ft.* 6 *in., find the cost of papering at* 5*s.* 6*d. per roll* (12 *yds. by* 21 *in.*). *Allow one roll for waste.*

$$\begin{aligned} &\text{Area of walls} = 680 \text{ sq. ft.} \\ &\text{Area of doors, etc.} = (28 + 48 + 27) = 103 \text{ sq. ft.} \\ \therefore\ &\text{Area to be papered} = 577 \text{ sq. ft.} = 64\tfrac{1}{9} \text{ sq. yds.} \end{aligned}$$

Area of 1 roll $= (\frac{21}{36} \times 12) = 7$ sq. yds.

$\therefore$ No. of rolls $= \dfrac{64\frac{1}{9}}{7} + 1 = 11.$

$\therefore$ Cost of papering $= 5s.\ 6d. \times 11 = £3\ 0s.\ 6d.$

69. *Floor coverings.*—

Example 3.—*What length of linoleum* 2 *yds. wide is required to cover the floor of a room, the dimensions of which are: length* 20 *ft.* 6 *ins., width* 18 *ft.? Allow* $1\frac{1}{2}$ *yds. for waste in fitting.*

Area of room $= \frac{20\frac{1}{2} \times 18}{9}$ sq. yds.
$= 41$ sq. yds.

Area of 1 yd. of lino $= (1 \times 2)$ sq. yds.

$\therefore$ No. of yds. required $= \frac{41}{2} = 20\frac{1}{2}$.

Adding $1\frac{1}{2}$ yds. waste, total $= 20\frac{1}{2} + 1\frac{1}{2} = 22$ yds.

Example 4.—*The centre of a carpet is formed by sewing together six strips each* 20 *ft. long, cut from a roll* 30 *ins. wide and costing* 7*s.* 9*d. per yd. A border* 18 *ins. deep is then fitted round the whole. If the cost of the border is* 5*s.* 6*d. per yd., find the total cost of carpet used, allowing* 2 *yds. of centre and* 1 *yd. of border for waste in matching.*

Dimensions of centre of carpet:

Length $= 20$ ft.
Width $= (2\frac{1}{2} \times 6) = 15$ ft.

Dimensions of whole carpet:

Length $= (20 + 3)$ ft. $= 23$ ft.
Width $= (15 + 3)$ ft. $= 18$ ft.

$\therefore$ * Length of border $= 2(23 + 18)$ ft.
$= 82$ ft.

* The length of border required is given by the outside perimeter, not by the inside perimeter, since in order to effect a joining at the corners portions are cut away. The student should observe the difference between this problem and Ex. 3 on page 78.

Amount of border required = 82 ft. + 1 yd. waste = 85 ft.
Cost of border = 85 ft. @ 5/6 per yd. = £7 15*s*. 10*d*.
Amount of centre reqd. = (20 × 6) ft. + 2 yds. = 42 yds.
Cost of centre = 42 yds. @ 7/9 per yd. = £16 5*s*. 6*d*.
Total cost = £7 15*s*. 10*d*. + £16 5*s*. 6*d*. = £24 1*s*. 4*d*.

Examples IX*b*

(1) Find the cost of papering the walls of the following rooms if the paper is in rolls 12 yds. long by 21 ins. wide, and given the dimensions, etc., are (allow one roll waste in each case):

(*a*) Length 18 ft. 9 in., breadth 15 ft. 6 in., height 12 ft., space for doors, etc. 100 sq. ft., price per roll 2*s*. 6*d*.

(*b*) Length 22 ft., breadth 16 ft. 9 in., height 10 ft. 6 in., space for doors, etc. 78 sq. ft., price per roll 3*s*. 4*d*.

(*c*) Length 20 ft. 3 in., breadth 18 ft. 6 in., height 10 ft., space for doors, etc. 140 sq. ft., price per roll 5*s*. 6*d*.

(*d*) Length 23 ft. 4 in., breadth 14 ft. 8 in., height 12 ft., space for doors, etc. 96 sq. ft., price per roll 4*s*. 10*d*.

(*e*) Length 16 ft. 8 in., breadth 12 ft. 9 in., height 10 ft. 6 in., space for doors, etc. 72 sq. ft., price per roll 3*s*. 8*d*.

(2) Find the cost of distempering the following rooms, the dimensions of door, window, and fireplace being given separately:

(*a*) Length 21 ft. 4 ins., breadth 16 ft. 8 ins., height 12 ft., door 4 ft. by 7 ft., window 4 ft. by 6 ft., fireplace 4 ft. 6 ins. by 6 ft., cost per sq. yd. 9*d*.

(*b*) Length 38 ft. 6 ins., breadth 22 ft. 9 ins., height

11 ft., door 6 ft. by 8 ft., window 10 ft. by 5 ft., fireplace 5 ft. by 6 ft., cost per sq. yd. 6*d.*

(*c*) Length 16 ft. 8 ins., breadth 12 ft. 3 ins., height 10 ft., door 4 ft. by 7 ft., window 3 ft. by 7 ft., fireplace 5 ft. by 5 ft., cost per sq. yd. 8*d.*

(3) The walls of a room 28 ft. long, 20 ft. 6 ins. wide, and 14 ft. high have a frieze 2 ft. deep around their upper edge. If doors, etc., occupy 120 sq. ft., find the cost of papering the remainder, given that rolls 12 yds. long by 27 ins. wide cost 7*s.* 10*d.* each. Allow one piece for waste.

(4) In the above example find the cost of the frieze at 13*s.* 9*d.* per roll of 9 yds., if the frieze can be cut to the half-yard.

(5) A room 40 ft. long and 28 ft. wide has a dado placed round it to a height of 3 ft. This is broken by a door 4 ft. and a fireplace 6 ft. wide. Find the total cost if a roll of the dado 9 yds. long and 24 ins. wide costs 14*s.* 6*d.* Allow $1\frac{1}{2}$ yds. waste for fitting.

(6) The upper portion of the walls in Example 5 is surrounded by a frieze 15 in. deep. Find the cost if the frieze is sold in panels 37 in. long at 4*s.* 6*d.* per panel. Allow one panel waste.

(7) A poultry run 100 yds. long by 80 yds. wide is enclosed with wire netting to a height of 6 ft. Find the cost if the wire netting is priced at 30*s.* 4*d.* per roll 50 yds. long by 6 ft. wide. Assume that the netting can be cut to the yard.

(8) What would have been the cost if in the above example the netting had been bought in rolls 3 ft. wide at 15*s.* 6*d.* per roll ?

(9) A border 18 in. wide is placed round the ceiling of a room 27 ft. long by 22 ft. wide, while the centre is covered with imitation panelling. Find the cost if the border is priced at 6*s.* 9*d.* per

yd. and the centre is bought at the rate of 12*s*. 6*d*. per roll 9 yds. long by 24 ins. wide. Allow 1 yd. waste for the centre, 2 ft. for the border.

(10) A carpet is to be made for a floor 28 ft. long by 20 ft. 4 ins. wide, so as to leave a stained border showing to a width of 1 ft. round its edge. If the carpet itself contains a border 20 ins. wide, find how many yds. should be cut from a roll 30 ins. wide in order to build up the centre in the manner shown in Example 4, para. 68.

(11) A closed water-tank has dimensions: length 30 ft., width 20 ft., depth 16 ft. Find the cost of painting the exterior at 6*d*. per sq. yd. (Answer correct to nearest penny.)

(12) An open box is made of wood 1 in. in thickness so as to have its outer dimensions as follows: length 30 ins., width 24 ins., depth 18 ins. (*a*) Find the difference in area between the outside and inside surfaces (express in sq. ins.); (*b*) What is the cost of lining the inside of the box with lead at 3*s*. 6*d*. per sq. ft.?

In the succeeding examples use the following system of coinage:

10 mils = 1 cent.
10 cents = 1 florin.
10 florins = £1.

(All the answers should be given correct to the nearest cent.)

(13) Find the cost of distempering the walls of a room 5·7 m. long by 3·5 m. wide by 2·3 m. high at 5 cents 5 mils per sq. metre. Allow 9 sq. metres for doors, windows, etc.

(14) Find the cost of carpeting a room 6·3 m. long by 4·5 m. wide at 6 fl. 5 c. per sq. metre. Allow 1·5 sq. m. waste.

(15) Find the cost of papering the walls of a room 8·5 m. long by 6·8 m. wide and 4 m. high, making allowance for a door 2·3 m. by 1·2 m., two windows each 1·8 m. by 1·5 m., and a fireplace 2·5 m. wide by 1·6 m. high. The paper ·75 m. wide is sold in rolls 10 m. long at 2 fl. 5 c. per roll.

(16) A closed box, built of wood 3 cms. thick, has its outer dimensions 1·2 m., ·8 m., and ·6 m. respectively. Find the cost of lining the inside with lead at £1 3 fl. 5 c. per sq. metre.

70. Solid figures, with shape similar to that of an ordinary brick, are known as rectangular solids. If the length, breadth, and height are all equal, the figure is called a *cube*; if they are not equal, as in the case of a brick, beam of wood, etc., it is called a *cuboid.*

A cube is usually referred to by the length of any one of its edges; thus, a foot cube is one which has each of its edges a foot long. Its volume, or the amount of space enclosed within its faces, furnishes a unit of measurement, viz. the cubic foot, by means of which we can estimate the volume of other figures. In like manner we get other units, such as the cubic yard, cubic inch, cubic decimetre, etc.

71. The method of calculating the volume of any rectangular solid will best be understood by referring to one particular example.

Example : Find the cubical contents of a rectangular block of wood 12 *ins. long,* 8 *ins. broad, and* 5 *ins. high.*

(1) The block can be sawn into 5 flat boards, each 12 ins. long, 8 ins. wide, 1 in. thick.

(2) Each board can be sawn into 8 lengths, each 12 ins. long, 1 in. wide, 1 in. thick.

(3) Each length can be sawn into 12 cubes, each 1 in. long, 1 in. wide, 1 in. thick.

The total number of cubic inches, therefore = $12 \times 8 \times 5 = 480$, or, the volume of a rectangular block in cubic inches = no. of ins. in the length × no. of ins. in the width × no. of ins. in the height.

72. The above reasoning can be made perfectly general so as to cover any unit of measurement, with fractional as well as integral numbers.

The general formula is usually contracted to:

$$\text{Volume} = \text{length} \times \text{breadth} \times \text{height}.$$

This can also be written:

$$\text{Volume} = \text{area of base} \times \text{height};$$

and from this we get:

$$\text{Height} = \frac{\text{volume}}{\text{area of base}}; \quad \text{Area of base} = \frac{\text{volume}}{\text{height}}$$

In using these formulæ for purposes of calculation care must be taken to express each dimension in terms of the same unit.

Example 1.—*How many cubic yards of soil are taken out in digging the foundation for a building, if the excavation measures* 20 *yds. long,* 15 *yds. wide, and* 14 *ft. deep?*

$$\text{No. of cubic yds.} = (20 \times \overset{5}{\cancel{15}} \times \frac{14}{\underset{1}{\cancel{3}}}) = 1{,}400.$$

Example 2.—*A class-room has a ground area of* 1,200 *sq. ft. What must be the height of the room so as to allow* 300 *cubic ft. of air for each of* 60 *pupils?*

The room must contain (300×60) cubic ft.

Area of floor $= 1,200$ sq. ft.

$$\therefore \text{ Height of room } = \frac{\overset{1}{\cancel{300}} \times \overset{15}{\cancel{60}}}{\underset{4}{\cancel{1,200}}} = 15 \text{ ft.}$$

EXAMPLES IX*c*

(1) How many cubic feet of timber are contained in 100 planks, each 12 ft. by 9 ins. by 1 in. ?

(2) How many cubic feet of stone are there in a block 40 in. long, 30 ins. wide, and 12 ins. high ? What is the price at 2*s.* 6*d.* per cubic foot ?

(3) Find the cost of digging a ditch 120 yds. long, 4 ft. wide, and 30 ins. deep, at a cost of 5*s.* 6*d.* per cubic yard.

(4) How many 2-in. cubes can be cut from a block of wood 2 ft. long, 30 ins. wide, and 18 ins. deep ?

(5) A footpath 3 ft. 6 ins. wide and 80 yds. long is laid with paving stones 3 ins. thick. Find the total cost of stone used at 4*s.* 6*d.* per cubic foot.

(6) How many planks, each 12 ft. long, 9 ins. wide, and 1 in. thick, can be cut from a baulk of timber 36 ft. long, 21 ins. wide, and 18 ins. deep ?

73. ***Brickwork.***—A brick of usual size is $8\frac{1}{2}$ ins. long, 4 ins. broad, and $2\frac{1}{2}$ ins. thick, but allowing for the mortar when it is built into a wall, the dimensions are taken as 9 ins., $4\frac{1}{2}$ ins., and 3 ins. respectively.

Ordinary brickwork is estimated by the rod or sq. pole at a standard thickness of $1\frac{1}{2}$ bricks ($13\frac{1}{2}$ ins.). Since a sq. perch equals $(16\frac{1}{2} \times 16\frac{1}{2})$ or $272\frac{1}{4}$ sq. ft.,

a bricklayer's rod contains ($272 \times 1\frac{1}{8}$) or 306 cubic feet, the $\frac{1}{4}$ sq. ft. being neglected in calculation.

Example 1.—*A wall* 12 *ft. high and* 2 *bricks thick is built round a grass plot* 40 *ft. long and* 25 *ft. wide. Allowing for a doorway* 8 *ft. high by* 4 *ft.* 6 *ins. wide, find the cost of building at* £25 *per rod.*

The ground area of the brickwork can be obtained in the same manner as the area of the stained border in Example 3, para 65.

So that inside dimensions of wall = length 40 ft., width 25 ft.; outside dimensions of wall = length $(40 + 3)$ ft., width $(25 + 3)$ ft.

$\therefore$ Area covered by brickwork $= (43 \times 28)$ sq. ft. $-$ (40×25) sq. ft. $= (1{,}204 - 1{,}000)$ sq. ft. $= 204$ sq. ft.

Cubical contents of wall $-$ space for door $= (204 \times 12)$ cubic ft. $-$ $= (2{,}448 - 54)$ cubic ft. $= 2{,}394$ cubic ft.

No. of rods in wall $= \dfrac{2{,}394}{306}$

Cost of building $= \dfrac{£2{,}394 \times 25}{306}$ $= £195\ 11s.\ 9d.$

```
       195·588
  306 )59,850·
       29,25
        1,710
         1800
          270
           25
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EXAMPLES IX*d*

(1) How many bricks are there in a wall containing $2\frac{1}{2}$ rods? What is the cost of the bricks at £3 10*s*. per 1,000?

(2) How many rods of brickwork are there in a wall 2 bricks thick, 40 ft. long, and 10 ft. high?

(3) How many bricks are contained in a wall

$1\frac{1}{2}$ bricks thick, 36 ft. long, and 12 ft. high? How many cubic ft. of mortar are there? (See para. 73.)

(4) What length of wall $1\frac{1}{2}$ bricks thick and 8 ft. high can be built from a stack containing 3,600 cubic ft. of bricks? (See para. 73.)

(5) In Example 1, para. 72, find the cost of lining the excavation with bricks to a thickness of $1\frac{1}{2}$ bricks, given the cost per rod as £27 10*s.*

(6) In the above example find the cost of the actual bricks used at £3 10*s.* per 1,000.

(7) A wall 64 ft. long, 12 ft. high, and 18 ins. thick is to be built of Bath stone at an inclusive cost of 3*s.* 3*d.* per cubic ft. What saving would be effected if the wall were built of bricks at £26 10*s.* per rod?

74. The specific gravity of a body is the ratio of its weight to the weight of an equal volume of water—e.g. the specific gravity of lead is 11·4, or in non-technical language, lead is 11·4 times as heavy as water.

A cubic foot of water weighs 1,000 ounces.

1 gallon of water weighs 10 lbs.

Example 1.—*How many gallons of water are contained in a tank* 10 *ft. long,* 8 *ft. wide, and* 5 *ft. high?*

$$\text{Volume of tank} = (10 \times 8 \times 5) \text{ cubic ft.}$$
$$\text{Weight of water} = (400 \times 1{,}000) \text{ ozs.}$$
$$\text{No. of gallons} = \frac{400 \times 1{,}000}{10 \times 16} = 2{,}500.$$

Example 2.—*An open rectangular box made of wood* 1 *in. thick has its outer dimensions: length* 30 *ins., breadth* 20 *ins., depth* 10 *ins. If the specific gravity of the wood is* ·65, *find to what depth the box*

will sink in water, given that a floating body displaces its own weight of liquid.

Outer dimensions of box: length 30 ins., breadth 20 ins., depth 10 ins.

Inner dimensions of box: length 28 ins., breadth 18 ins., depth 9 ins.

Volume of wood:

$$= (30 \times 20 \times 10) \text{ cubic ins.} - (28 \times 18 \times 9) \text{ cubic ins.}$$
$$= (6{,}000 - 4{,}536) \text{ cubic ins.}$$
$$= 1{,}464 \text{ cubic ins.}$$

This is equivalent in weight to $(1{,}464 \times {\cdot}65) = 951{\cdot}6$ cubic ins. of water.

Area of bottom of box = 600 sq. ins.

$\therefore$ Box sinks to a depth of $\frac{951{\cdot}6}{600} = 1{\cdot}586$ in., or $1{\cdot}6$ in. (approx.).

Examples IX*e*

(1) Find the holding capacity of a closed wooden box, given that the outer dimensions are: length 2 ft. 6 ins., breadth 2 ft., and height 20 ins. Thickness of the wood is $\frac{1}{2}$ in.

(2) Find the weight of the above box, given the specific gravity of the wood equals ·84.

(3) Given that the specific gravity of ice is ·92, find the weight of a block of ice 30 in. long, 20 in. wide, and 18 in. deep. (Correct to nearest lb.)

(4) A reservoir is 300 ft. long, 240 ft. wide, and 12 ft. deep. What is its holding capacity in gallons?

(5) What weight of water falls per sq. mile for a rainfall of 1 in.? (Correct to the nearest ton.)

(6) A reservoir 250 yds. long, 170 yds. wide, and 20 ft. deep drains an area of 1·5 sq. miles. How many inches of rainfall are needed to fill the

reservoir, assuming that only one-third of the water drains into it ?

(7) Find, correct to the nearest lb., the weight of a closed zinc cistern full of water, if the outer dimensions of the cistern are 2 ft. 6 ins. by 2 ft. by 1 ft. 6 ins., the thickness of the zinc being $\frac{1}{12}$th of an inch and its specific gravity 7·1.

(8) Given that sheet lead costs £40 per ton and its specific gravity is 11·4, find the cost per sq. ft. of sheet lead $\frac{1}{8}$ in. thick.

For the undermentioned examples it should be remembered that 1 c.c. of water weighs 1 gram ; 1 litre of water = 1 cubic decimetre of water, weighs 1,000 gms. ; 1 tonne = 1,000 kg.

(9) Find the holding capacity of a reservoir in litres, given the dimensions are : length 105 m., width 45 m., depth 6·5 m.

(10) (*a*) Find the weight of water which falls per hectare during a rainfall of ·75 cm.

(*b*) How many tanks, each 3 m. long, 2 m. wide, and 1·5 m. high, could be filled by the above volume of water ?

(*c*) If one-third of the water drained into a reservoir 100 m. long by 40 m. wide, find the corresponding rise in surface of the reservoir. (Correct to mm.)

(11) A plank of wood 4 m. long × 25 cm. × 4 cm. is held upright in water. Find its weight if it sinks to a depth of 2·6 m. (Note, the plank displaces its own weight of water.)

CHAPTER X

UNITARY METHOD OF PROPORTION

75. THIS method of working proportion may be readily understood by the consideration of a few examples.

EXAMPLES

1. *If* 24 *articles cost* £4 4*s*., *find the cost of* 56 *such articles.*

24 articles cost 84*s*.

1 article costs $\frac{84}{24}s.$

$$\therefore 56 \text{ articles cost } \frac{\overset{28}{\cancel{84}}}{\underset{\cancel{3}}{\cancel{24}}} \times \frac{7}{\cancel{56}}s. = 196s. = £9\ 16s.\ 0d.$$

2. *If* 24 *men complete a piece of work in* 84 *days, how long will it take* 56 *men ?*

24 men complete the work in 84 days.

Note.—1 man completes work in 84 × 24 days.

$$\therefore 56 \text{ men complete work in } \frac{\overset{12}{\cancel{84}} \times \overset{3}{\cancel{24}}}{\underset{\cancel{7}}{\cancel{56}}} \text{ days} = 36 \text{ days.}$$

Example 1 illustrates the direct proportion, and the working of each step can readily be seen and

understood. Example 2 illustrates the inverse method—the variety of such examples is small.

The first statement must be so arranged that the quantity of the same kind as the answer comes last.

EXAMPLES X*a*

(1) If 72 articles cost £4 10*s*., find the cost of 12.

(2) If 65 tons are carried for £4 11*s*., what should be charged for 55 tons ?

(3) What will be cost of 18 yds. of cloth if 32 yds. cost £2 8*s*. ?

(4) By travelling at 24 miles an hour a journey is completed in 5 hours. How long should the journey take travelling at 36 miles per hour ?

(5) A quantity of material lasts 75 workmen for 33 days. How long should the same quantity last 45 workmen ?

(6) If I earn £8 10*s*. in 12 days, how much should I earn in 28 days ?

(7) If I earn £8 10*s*. in 12 days, how long will it take me to earn £59 10*s*. ?

(8) How far will a train travel in 16 minutes at the rate of 24 miles per hour ?

(9) If I cycle at 8 miles per hour, how many yards do I cover in one minute ?

(10) A man spends £38 12*s*. 6*d*. in 30 days. How long will it take him to spend £10 6*s*. at same rate ?

(11) If $\frac{3}{8}$ of a ton of coal costs 15*s*., how much is this per ton ? What will $\frac{2}{5}$ of a ton cost ?

(12) If $\frac{5}{6}$ of the profits of a firm amount to £3,846, what will $\frac{5}{8}$ of profits amount to ?

(13) Taking 1 Kilometre = $\frac{5}{8}$ mile, if to lay a road costs £250 per Km., what should it cost per mile ?

(14) Taking 1 hectare = $2\frac{1}{2}$ acres, what should be cost per acre if $37\frac{1}{2}$ hectares cost 787 fr. 50 c.?

(15) £845 16*s.* converted into French money = 21,314 fr. 16 c. Find to the nearest penny the equivalent of 7,053 fr. 35 c.

76. Compound Proportion may be most easily worked by the unitary method.

Example

If 72 *tons are carried* 54 *miles for* £81, *how far should* 80 *tons be carried for* £96?

For £81 72 tons are carried 54 miles.

,, £1 72 tons are carried $\frac{54}{81}$ miles.

,, £1 1 ton is carried $\frac{54}{81} \times \frac{72}{1}$ miles.

,, £96 1 ton is carried $\frac{54}{81} \times \frac{72 \times 96}{1}$ miles.

,, £96 80 tons are carried $\frac{\overset{6}{\cancel{54}}}{\underset{9}{\cancel{81}}} \times \frac{\overset{9}{\cancel{72}} \times 96}{\underset{10}{\cancel{80}}}$ miles.

$$= \frac{576}{10} = 57{\cdot}6 \text{ miles.}$$

Examples X*b*

(1) If the wages of 36 men amount to £234 in three weeks, how long will it take for the wages of 30 men to amount to £165?

(2) If 16 men plough 120 acres in 15 days, how many men would be required to plough 200 acres in 16 days?

(3) If 45 bushels feed 60 horses for one week, how many days would 72 bushels feed 48 horses?

(4) If 35 men build a wall 147 ft. long in 21 days, how many men would be required to build a wall 84 ft. long in 28 days?

(5) If 8 people require £260 to live on for 52 weeks, how long could 12 people live on £390?

(6) One hundred and twelve tons are carried 65 miles for £49. How many tons would be carried 48 miles for £42?

PROPORTIONAL PARTS AND PARTNERSHIP

77. A partnership is the combination of two or more persons (not exceeding twenty) carrying on business for profit. Each partner brings to the business some amount of capital, or possibly some particular knowledge, ability, or experience. The share of the profit to which each partner is entitled, and all the conditions of the partnership, are drawn up and embodied in the "Articles of Partnership" which each partner must sign. Where some are active and some "sleeping" partners, the proportion of profit to which each is entitled is a matter for mutual arrangement, but where all partners share equally in the working of the firm the division of the profits is generally in proportion to the capital which each has brought into the business. Sometimes each partner draws a salary, after which the profits are divided.

EXAMPLES

1. *A, B, and C are partners in business, their capitals being* £1,200, £2,000, *and* £3,000 *respectively. The profits, which are to be divided in pro-*

portion to the capitals, amount to £930. What is the share of each?

The total capital is £(1,200 + 2,000 + 3,000) = £6,200.

A's capital is $\frac{1200}{6200}$ of total capital $= \frac{6}{31}$
B's ,, ,, $\frac{2000}{6200}$,, ,, $= \frac{10}{31}$
C's ,, ,, $\frac{3000}{6200}$,, ,, $= \frac{15}{31}$

∴ Share of A is $\frac{6}{31}$ of £930 = £180.
,, ,, B ,, $\frac{10}{31}$ of £930 = £300.
,, ,, C ,, $\frac{15}{31}$ of £930 = £450.

2. *Three partners, A, B, and C, with capitals of £720, £1,080, and £1,200 engage in business. Before dividing the profits, which amount to £900, A draws a salary of £200 as manager and B £160 as cashier. What is each partner's share?*

Total capital = £3,000.
Profits = £900 − £360 = £540.

A's share $= \frac{720}{3000} = \frac{6}{25}$ of £540 = £129 12*s*. + £200 = £329 12*s*.

B's share $= \frac{1080}{3000} = \frac{9}{25}$ of £540 = £194 8*s*. + £160 = £354 8*s*.

C's share $= \frac{1200}{3000} = \frac{2}{5}$ of £540 = £216.

3. *A and B are partners with capitals of £2,500 and £3,500 respectively. After being in business for three months, C enters into partnership with them, bringing in a capital of £2,000. What should be each one's share of the profits, which amount to £1,750?*

Here the claims of each partner depend upon the time each capital was employed as well as the amount of capital.

Consider the employment of £1 for one month as unit.

£2,500	for 12	months	= 30,000	units.
£3,500	,, 12	,,	= 42,000	,,
£2,000	,, 9	,,	= 18,000	,,

Total = 90,000 units.

A's share $= \frac{30000}{90000} = \frac{1}{3}$ of £1,750 = £583 6*s*. 8*d*.
B's ,, $= \frac{42000}{90000} = \frac{7}{15}$ of £1,750 = £816 13*s*. 4*d*.
C's ,, $= \frac{18000}{90000} = \frac{1}{5}$ of £1,750 = £350.

Examples Xc

(1) A's capital = £400, B's £600, C's £800, profits £450. What is the share of each?

(2) A's capital = £1,600, B's £2,400, C's £4,000, profits = £1,730. What is the share of each?

(3) A, B, C, and D enter into partnership. A brings no capital, but takes $\frac{1}{5}$th share of profits for salary as manager. The capitals of B, C, and D are £725, £1,250, and £2,000 respectively. What would each obtain if profits amount to £1,060?

(4) Two partners, X and Y, trade for six months and then admit into partnership Z. X's capital is £4,500, Y's £5,000, Z brings £6,500. The profits amount to £2,040. What is share of each?

(5) Four partners, with capitals of £6,000, £4,800, £4,200, and £3,000 respectively, are engaged in business. After three months the first partner draws £1,200 from his capital. What share should each have of the profits, which amount to £3,078?

(6) Three partners, A, B, and C, are in business with capitals of £2,500, £3,500, and £4,000 respectively. At the end of four months they each increase their capitals by £500, and bring D into

partnership with capital of £3,000. What should be the share of each, if the profits amount to £2,262?

(7) Three farmers graze oxen in a field and share the rent according to the use they make of the field. One grazes 16 oxen for a total of six months, one 20 for four months, and the third 24 for two months. How should they divide the rent, which is £28?

CHAPTER XI

PERCENTAGES

79. WHENEVER two or more fractions are to be compared, they must first of all be reduced to a common denominator. By the ordinary method this would vary with each group of fractions, so that it is found more convenient to choose a standard denominator to which all others can be reduced. The standard chosen is 100—and any fraction written with this denominator is called a *percentage*—while its numerator is termed the *rate per cent.*

80. The following figures illustrate the use of percentages as forming a means of comparison between different ratios.

Railway.	Gross Receipts.	Working Expenses.	Percentage of Working Expenses to Gross Receipts.
Rhymney .	£440 thousands	£284 thousands	65
South-Eastern & Chatham	£6,060 thousands	£4,057 thousands	67
Taff Vale .	£1,321 thousands	£885 thousands	67

The first ratio $\frac{\text{working expenses}}{\text{gross receipts}}$ can be written as $\frac{284}{440}$, or reducing its denominator to 100, as $\frac{65}{100}$. Similarly the other two ratios $\frac{4057}{6060}$ and $\frac{885}{1321}$ can each be rewritten as $\frac{67}{100}$. In actual practice, however, the numerators only are written, the

fractional significance of the figures being indicated by the use of the symbols % or p.c. Thus the first example would be written either as 65% or 65 p.c., and read off as 65 per cent.

The above rates per cent. are only approximately correct, as usual when such fractions are reduced to percentages. The degree of accuracy to be adopted is largely a matter to be decided for individual cases.

81. To rewrite any fraction as a percentage is simply equivalent to expressing its value in terms of hundredths, e.g. :

Write as percentages (a) $\frac{17}{20}$, (b) $\frac{1136}{1978}$.

Since $1 = \frac{100}{100}$ or 100 hundredths.

Then $\frac{17}{20} = \frac{17}{20}$ of $1 = \frac{17}{20}$ of 100 hundredths.

$= \frac{17 \times 100}{20}\% = 85\%$.

Similarly $\frac{1136}{1978} = \frac{1136 \times 100}{1978}\% = 57 \cdot 4\%$.

Therefore: *To express any fraction as a percentage, multiply its numerator by* 100 *and divide the result by the denominator.*

Example 1.—*The annual cost of running an electric delivery car is* £178 5*s*. *The item for tyres amounts to* £14 15*s*. *What percentage of the total cost does this represent?* (*Correct to first decimal place.*)

Percentage $= \frac{14\frac{3}{4} \times 100}{178\frac{1}{4}} = \frac{5900}{713} = 8 \cdot 3\%$.

Example 2.—*The average price of wheat in* 1901 *was* 26*s*. 9*d*. *per quarter. Find the percentage increase if in* 1902 *the price had risen to* 28*s*. 1*d*. *per quarter.*

Actual increase in price = 28*s*. 1*d*. − 26*s*. 9*d*. = 1*s*. 4*d*.

∴ Percentage increase $= \frac{1/4 \times 100}{26/9} = \frac{1600}{321} = 5\%$.

N.B.—In reckoning increase or decrease per cent. the calculation is based upon the *original* figures, so that the fractional increase is $\frac{1/4}{6/9}$ not $\frac{1/4}{28/1}$.

EXAMPLES XI*a*

Work the following correct to the first decimal place:

(1) The world's petroleum output in 1916 was 461 million barrels. Of this the U.S.A. produced 300 million barrels. What percentage of the whole did the rest of the world produce?

(2) By how much per cent. is the price of coal @ 2*s*. 8*d*. per cwt. dearer than the price @ 45*s*. 6*d*. per ton.

(3) In 1918 the duty on tobacco was raised from 6*s*. 5*d*. to 8*s*. 2*d*. per lb. What was the percentage increase?

If in 1919 the duty had dropped again to its original value, what percentage decrease would this have represented?

(4) If a firm's gross receipts were £3,654 and the expenses £2,832, what percentage of the receipts does the profit represent?

(5) The number of books published in 1917 was 8,131 as against 9,149 in 1916. Find the decrease per cent.

(6) In 1917 the applications for patents numbered 19,285, being a decrease of 683 on the number for 1916. What was the decrease per cent.?

(7) The price of wheat per quarter for the years 1913–17 was 31*s*. 8*d*., 34*s*. 11*d*., 52*s*. 10*d*., 58*s*. 5*d*., and 70*s*. 8*d*. respectively. Find the percentage increase each year.

(8) The national capital before the war was estimated at £15,019 millions. Transport, industries, etc., absorbed £3,753 millions, and agri-

culture £876 millions. What percentage did each form of the whole.

82. Since a percentage is written essentially for purposes of comparison it is not always the most convenient *working* form that a fraction can take; thus, it is easier to work with $\frac{1}{3}$ than its equivalent form $33\frac{1}{3}$ per cent.

In some instances, therefore, when a percentage is quoted we may wish to reduce it to its simplest fractional form. This is done by writing the rate per cent over 100, and reducing the resulting fraction to its lowest terms, e.g.:

$$33\tfrac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{100}{300} = \frac{1}{3}$$

83. The following simple fractional equivalents should be remembered:

$50\% = \frac{1}{2}$	$33\frac{1}{3}\% = \frac{1}{3}$	$25\% = \frac{1}{4}$	$20\% = \frac{1}{5}$
$16\frac{2}{3}\% = \frac{1}{6}$	$12\frac{1}{2}\% = \frac{1}{8}$	$10\% = \frac{1}{10}$	$8\frac{1}{3}\% = \frac{1}{12}$
$5\% = \frac{1}{20}$	$3\frac{1}{3}\% = \frac{1}{30}$	$2\frac{1}{2}\% = \frac{1}{40}$	$2\% = \frac{1}{50}$

Examples XI*b*

Using the equivalent fractional forms for the percentages, work the following mentally:

(1) $12\frac{1}{2}\%$ of (*a*) £24, (*b*) 1 cwt., (*c*) 1 mile, (*d*) 1 bushel, (*e*) 3 sq. ft.

(2) 10% of (*a*) £45, (*b*) 1 ton, (*c*) 1 furlong, (*d*) 1 gall. 1 qt., (*e*) 1 acre.

(3) 5% of (*a*) £1 5*s*., (*b*) 2 qrs. 14 lbs., (*c*) 2 yds. 8 ins., (*d*) 1 peck 2 qts., (*e*) 6 sq. yds. 6 sq. ft.

(4) $3\frac{1}{3}\%$ of (*a*) £9, (*b*) 1 cwt. 68 lbs., (*c*) 1 furlong 80 yds., (*d*) 1 pk. 1 gall. 3 qts., (*e*) 1 acre 20 sq. yds.

(5) $8\frac{1}{3}\%$ of (*a*) £1 4*s*., (*b*) 2 qrs. (*c*) 1 mile 4 furlongs, (*d*) 3 galls., (*e*) 3 acres.

Example 1.—*A traveller earns* $2\frac{1}{2}\%$ *commis-*

sion on his sales. What does this amount to on £648 16*s*. ?

$$2\tfrac{1}{2}\% \text{ on } £648\ 16s. = \frac{£648\ 16s.}{40} = £16\ 4s.\ 5d.$$

Example 2.—*A broker charges* $\frac{1}{8}\%$ *on his purchases. What is his commission on* £4,800 ?

$$\begin{aligned} \text{Commission} &= £\tfrac{1}{8} \text{ on each } £100. \\ &= £\tfrac{1}{8} \times 48 = £6. \end{aligned}$$

Example 3.—*Find the value of* $7\frac{1}{3}\%$ *of* £396 15*s*. 8*d*. (*Correct to nearest penny*.)

$$\begin{aligned} \text{Value} &= \frac{£396{\cdot}783 \times 7\frac{1}{3}}{100} \\ &= £29{\cdot}097 \\ &= £29\ 1s.\ 11d. \end{aligned}$$

$$\begin{array}{r|l} & £3{\cdot}9678\,|\,3 \\ & 7\tfrac{1}{3} \\ & 27{\cdot}7748 \\ \tfrac{1}{3} = & 1{\cdot}3226 \\ & £29{\cdot}0974 \end{array}$$

Examples such as the above should be worked by decimalising the money and moving the decimal point two places to the left to divide by 100. The fractional method of working should only be retained in cases where the percentage cancels down to a very easy fraction as in 1 and 2:

Examples XI*c*

(1) Work out the following commissions (correct to the nearest penny):

(*a*) 3% on £350. (*b*) $2\frac{1}{2}\%$ on £272. (*c*) 7% on £284. (*d*) 6% on £649 15*s*. (*e*) 4% on £85 16*s*. (*f*) $3\frac{1}{2}\%$ on £341 17*s*. (*g*) $4\frac{1}{2}\%$ on £493 16*s*. 9*d*. (*h*) $3\frac{1}{2}\%$ on £93 18*s*. 3*d*. (*i*) $5\frac{1}{2}\%$ on £81 16*s*. 8*d*.

(2) A house previously rented at £55 per year has its rent increased by 8%. Find the new rent.

(3) Milk is found to be watered to the extent of 14%. How much water has been added to 9 galls. 2 qts. of pure milk ? (Correct to nearest pint.)

(4) What is the purchasing value of a sovereign (in shillings and pence) compared with its pre-war value, if the cost of commodities has increased by 108%?

(5) A person who bought a house in 1914 for £346 wishes to sell again in 1919 so as to be in relatively the same position. What price must he now ask if he finds the cost of living has increased by 112%? (Correct to nearest £.)

(6) A man's pre-war salary was £145 per annum. It is now revised so as to improve his condition by 20%. What must his new salary be if the purchasing value of a sovereign has dropped from 20*s*. to 9*s*. 3*d*.? (Correct to nearest £.)

(7) A man's salary increases from £135 to £235, but meanwhile the cost of living has also increased by 105%. What is the true gain or loss per cent. in the value of his salary?

(8) A book published in 1914 at 1*s*. 6*d*. per copy cost $10\frac{1}{2}$*d*. to produce. The publisher revises the price in order to meet the decreasing purchasing value of a sovereign and the increased cost of production. Given that the former has decreased by 53% and the latter increased by 135%, find the revised price correct to the nearest penny.

(9) An article which costs 2*s*. 1*d*. to produce is sold for 3*s*. 6*d*. What was its former equivalent price, if the purchasing value of a sovereign has decreased from 20*s*. to 9*s*. 7*d*., while the cost of producing the article has risen by 90%?

(10) A man's holiday expenses used to work out as follows: travelling £2 5*s*., rooms £5 5*s*., food £3 10*s*., incidentals £2 10*s*. He now finds the various items have increased by 50%, 100%, 120%, and 70% respectively. What is the average percentage increase? (Correct to one decimal place.)

84. When converting fractions having a large number of digits to percentage form, contracted methods of division should be used.

Example 1.—*The working expenses of the G.W.R. in* 1909 *were* £13,210,438, *while the gross receipts were* £18,810,744. *Find the percentage of the former to the latter.* (*Correct to one decimal place.*)

Percentage

$$= \frac{13210438 \times 100}{18810744}$$

$$= \frac{132\cdot10438}{1\cdot8810744}$$

$$= 70\cdot2$$

Rough check :

$$\frac{132}{1\cdot9} = 70$$

$$\begin{array}{r|r|}
 & 70\cdot22 \\
1\cdot\cancel{8}\cancel{8}\cancel{1}\cancel{0}\cancel{7}\cancel{4}\cancel{4} & 132\cdot10 \\
 & 131\ 67 \\ \hline
 & 43 \\
 & 38 \\ \hline
 & 5 \\
 & 4
\end{array}$$

Example 2.—*What percentage is* £3 17*s*. 5*d*. of £8 11*s*. 4*d*? (*Correct to one decimal place.*)

£3 17*s*. 5*d*. = £3·8708$\dot{3}$
£8 11*s*. 4*d*. = £8·5$\dot{6}$

Percentage

$$= \frac{3\cdot8708\dot{3} \times 100}{8\cdot5\dot{6}}$$

$$= \frac{387\cdot08\dot{3}}{8\cdot5\dot{6}}$$

$$= 45\cdot2$$

Rough check :

$$\frac{380}{8} = 47$$

$$\begin{array}{r|r|}
 & 45\cdot18 \\
8\cdot\cancel{5}\cancel{6}\cancel{6}\cancel{6} & 387\cdot08 \\
 & 342\ 67 \\ \hline
 & 4441 \\
 & 4283 \\ \hline
 & 158 \\
 & 86 \\ \hline
 & 72 \\
 & 68 \\ \hline
 & 4
\end{array}$$

EXAMPLES XI*d*

Work the following correct to 1 *place of decimals:*

(1) Fill in column (*d*), Examples I*a*, No. 1, (i) to (x).

(2) In the year 1915–16 income tax was paid on £873,841,065. If the revenue produced amounted to £118,765,226, what was the average rate per cent.?

(3) The receipts into Exchequer for the years 1913-18 were as follows: £198·243 millions, £226·694 millions, £336·767 millions, £573·428 millions, and £707·235 millions respectively. What was the percentage increase each year?

(4) The imports for the U.K. in 1916 were £948,506,492, and in 1917 £1,064,164,678. Find the percentage increase.

(5) A person finds that his income from various investments amounts to £87 17*s.* 5*d.* per annum. What average rate per cent. does this represent if the investments amount to £1,325 10*s.* 6*d.*

(6) A person receives 4½% on £86 10*s.*, 5% on £261 5*s.*, 6% on £315 13*s.* 4*d.*, and 5½% on £542 6*s.* 8*d.* What is the average rate per cent. earned?

CHAPTER XII

PROFIT AND LOSS

85. TRADE is carried on with the object of earning a profit, and the success or non-success of a business is gauged by the amount of profit gained.

Profit is generally expressed as a percentage of the original outlay, i.e. of the cost price. Many firms base the calculation of the percentage of profit upon the selling price.

Thus, if a man buys goods for £10 and sells for £15, his profit is £5 on £10, i.e. 50 per cent.

Business men claim that of the £15 received, £5 is profit, and this they say is $33\frac{1}{3}$ per cent.

However, there is no justification for the latter method except that in trade it has been found the more convenient.

In the following exercise, unless otherwise stated, the profit is to be reckoned on the cost price.

To calculate the profit or loss per cent.:

Example 1.—*A man buys goods for* £12 *and sells for* £15. *Find gain per cent.*

On £12 he gains £3

$\therefore$ „ £100 „ „ $\frac{3}{12} \times \frac{100}{1} = 25$ *per cent.*

Example 2.—*A man buys goods for* £15 *and sells for* £12. *Find loss per cent.*

On £15 he loses £3

$\therefore$ „ £100 „ „ $\frac{3}{15} \times 100 = 20$ *per cent.*

Example 3.—A man wishes to gain 15 per cent. on goods purchased for £16. At what price must he mark them?

Required selling price = 115 per cent. of C.P.

100 per cent. = £16

115 ,, $= £\frac{16}{100} \times \frac{115}{1} = £\frac{92}{5} = £18\ 8s.$

Example 4.—Goods marked £28 bear a profit of 40 per cent. of cost price. What was original cost?

Marked price = 140 per cent. of cost price.

140 per cent. = £28

∴ 100 ,, $= \frac{28}{140} \times 100$

= £20

Examples XIIa

Find gain or loss per cent. from following figures:

(1)	Cost price	£15,	selling price	£18.
(2)	,,	£28	,,	£35.
(3)	,,	2*s*. 6*d*.	,,	3*s*.
(4)	,,	£10	,,	£9 10*s*.
(5)	,,	£3 10*s*.	,,	£4 4*s*.
(6)	,,	£55	,,	£52 5*s*.

Find the selling price from following:

(7)	Cost price,	£7 15*s*.	gain	15	per cent.
(8)	,,	£5 12*s*.	,,	12½	,,
(9)	,,	£1 3*s*. 4*d*.	,,	40	,,
(10)	,,	£3 2*s*. 6*d*.	,,	3⅓	,,
(11)	,,	15*s*.	loss	2½	,,
(12)	,,	£140	,,	7½	,,
(13)	,,	£95	,,	8	,,
(14)	,,	£34 17*s*. 6*d*.	,,	6⅔	,,

Find cost price of article from following:

(15)	Sold for 21*s*.	at a profit of 5	per cent.
(16)	„ 34*s*. 6*d*.	„ „ 15	„
(17)	„ £31	„ „ $3\frac{1}{3}$	„
(18)	„ 19*s*.	at a loss 5	„
(19)	„ £1 1*s*.	„ „ 12	„
(20)	„ £2 1*s*. 6*d*.	„ „ $3\frac{3}{4}$	„

Mark the following goods to ensure the given profit:

(21)	Cost price 1*s*. $1\frac{1}{2}d.$,	profit $33\frac{1}{3}$	per cent.
(22)	„ 3*s*. 6*d*.	„ 25	„
(23)	„ 10*s*. 5*d*.	„ 4	„
(24)	„ £1 12*s*. 6*d*.	„ 15	„

(25) Soap costs 35*s*. per cwt. At what price must it be marked per lb. to obtain profit of 20 per cent.?

(26) At what price each must I mark articles costing 30*s*. per gross to obtain profit of 40 per cent.?

(27) A manufacturer clears $12\frac{1}{2}$ per cent. on goods costing him 25*s*. 4*d*. to make; the retailer makes a further $12\frac{1}{2}$ per cent. on the cost to him. At what price (nearest penny) will he mark the goods?

(28) A man buys 12 boxes of oranges, each containing 120, at 8*s*. 4*d*. per box. What profit does he make by selling at $1\frac{1}{2}d.$ each? What percentage is this?

(29) A dealer buys 6 dozen picture-frames for £5 10*s*. the lot. He sells 16 at 3*s*. each, 20 at 2*s*. 6*d*. each, 24 at 1*s*. 6*d*. each, and 12 at 1*s*. each. What was his gain per cent.?

(30) A chest of tea containing 36 lbs. costing 43*s*. 6*d*. is mixed with 12 lbs. at 1*s*. 10*d*. per lb. The mixture is sold at 2*s*. 3*d*. per lb. Find gain per cent. to nearest unit.

86. *Gross and Net Profits.*

Since in business these terms are frequently used, it would be well to obtain a clear idea of the difference between them and how each is obtained.

Gross profit is the amount by which the selling price of goods exceeds the cost price. Reckoned with the cost price are such charges as can reasonably be classed as a part of the cost of production. The gross profit is the balance of the Trading Account, a specimen of which is here given.

TRADING ACCOUNT FOR YEAR ENDED DECEMBER 31ST, 1918

Dr.				*Cr.*
	£		£	£
To Stock, January 1st, 1918.	2,540	By Sales	7,619	
,, Total Purchases £4,716		*Less* Returns	540	
,, *Less* Returns 357				7,079
	4,359	,, Stock, December 31st, 1918		3,164
,, Carriage and Freight	142			
,, Manufacturing Wages	1,347			
,, Gross Profit carried to Profit and Loss Account	1,855			
	£10,243			£10,243

The figures entered in this account are obtained from the various books which give full particulars of the details which produce the totals—the Purchases Book, Stock Book, Sales Book, Returns Book, Wages Book, etc. The gross profit is transferred to the credit side of the Profit and Loss Account and suffers deduction for all those expenses which, though necessary to the business, cannot reasonably be called a part of the cost of the production of the goods, but rather a part of the

cost of distribution, as office expenses, advertisement, etc. A specimen Profit and Loss Account is here given :

Profit and Loss Account for year ended December 31st, 1918

Dr.	£		*Cr.*	£
To Office Expenses .	79		By Gross Profit .	1,855
,, Salaries . .	465			
,, Rent, Rates, and Taxes . .	110			
,, Fuel and Lighting	40			
,, Bad Debts . .	28			
,, Net Profit . .	1,133			
	£1,855			£1,855

This net profit is added to the capital of the proprietor if a " one-man " firm, divided between the partners according to the agreement if a partnership, or distributed among the shareholders as dividend if a company.

Examples XII*b*

(1) Find the gross profit of the following: Stock at beginning of year, £1,638; purchases, £957, of which are returned £65; wages, £124; sales, £2,618; sales returns, £162; stock at end of year, £1,318.

(2) Find gross and net profit from the following figures obtained from the various books: Stock, January 1st, 1919, £3,547; purchases (less purchases returns), £1,472; wages, £218; sales (less sales returns), £2,035; stock, December 31st, 1919, £3,914; salaries, £210; office expenses, £63; bad debts, £54; rent, etc., £120.

(3) The books of Messrs. Cook & Co. show the following balances on December 31st, 1919:

Debtor balances: stock, January 1st, £3,010; purchases, £1,639; wages, £609; carriage, £65; office expenses, £94; salaries, £353; rent, etc., £110; bad debts, £140. Creditor balances: sales, £3,915; stock, December 31st, 1919, £2,400.

Make out Trading Profit and Loss Accounts and find net profit.

(4) The following are the balances of the book of The Carton Coal Co. on July 31st, 1919.

Debtor balances: stock, January 1st, 1919, £10,615; purchases, £5,318; wages, £1,796; carriage, £95; office expenses, £304; salaries, £854; rent, etc., £286; bad debts, £173; lighting, etc., £58; discount, £68. Creditor balances: sales, £8,176; stock, £12,744.

Note.—If on balancing the Trading Account the Debtor side exceeds the Creditor side, the balance is a gross loss, which is transferred to the Debit side of the Profit and Loss Account, and the amount by which that side exceeds the Credit side of the Profit and Loss Account is the net loss. Sometimes although there is a gross profit, the large expenses shown in the Profit and Loss Account are greater in amount and a net loss is made.

CHAPTER XIII

SIMPLE INTEREST

87. INTEREST is money paid for the use or loan of money. It is calculated at so much per year for each £100 lent, i.e. at so much per cent. per annum. The sum of money lent is called the Principal.

When interest is reckoned on the original principal only, throughout the period of the loan, it is termed "Simple Interest." Where the interest is added each year to make a new principal, the total interest for the term is called "Compound Interest." In this chapter "Simple Interest" only is dealt with.

There are several methods of calculating the simple interest, and the same exercise is here worked in two ways.

Example 1.—*To find the simple interest on £367 15s. 8d. for 3 years at 4 per cent.*

Unitary Method.—

Interest on : £

£100 for 1 year $= 4$

£1 for 1 year $= \dfrac{4}{100}$

£367 15*s*. 8*d*. for 1 year $= \dfrac{4}{100} \times 367\dfrac{47}{60}$

£367 15*s*. 8*d*. for 3 years $= \dfrac{\overset{2}{\cancel{4}}}{100} \times \dfrac{22067}{\underset{\underset{10}{\cancel{20}}}{\cancel{60}}} \times \dfrac{\cancel{3}}{1}$

$= \dfrac{44134}{1000}$

$=$ £44·134 $=$ £44 2*s*. 8*d*. to nearest penny.

(*Note.*—This method is easiest to understand, and is simplest to use where interest is required for an even number of years and where principal is an even number of pounds. It may be reduced to the formula:

$$\text{Interest} = \frac{\text{principal} \times \text{rate} \times \text{time}}{100}).$$

By decimal method.—

The interest on each £100 is £4. Number of times £100 is contained in the principal = £367·783 ÷ 100 = 3·67783.

To find interest multiply this by rate and time.

$$\begin{array}{r} \pounds \\ 3{\cdot}677\,83 \\ 4 \\ \hline 14{\cdot}711\,2 \\ 3 \\ \hline \pounds 44{\cdot}134 \end{array} = \pounds 44\ 2s.\ 8d.$$

Examples XIII*a*

By unitary method.—

Calculate interest being given:

(1) Principal = £325, rate 4 per cent, time 4 years.

(2) Principal = £715, rate 3 per cent., time 5 years.

(3) Principal = £625, rate $2\frac{1}{2}$ per cent., time 4 years.

(4) Principal = £475, rate $3\frac{1}{3}$ per cent., time 3 years.

(5) Principal = £968, rate 5 per cent., time 5 years.

(6) Principal = £624, rate $2\frac{1}{2}$ per cent., time $3\frac{1}{3}$ years.

By decimal method.—

(7) Find interest on £178 14*s.* 3*d.* for 4 years at 3 per cent.

(8) Find interest on £296 3*s.* 10*d.* for 6 years at $2\frac{1}{2}$ per cent.

(9) Find interest on £357 12*s.* 5*d.* for 5 years at 4 per cent.

(10) Find interest on £1,416 17*s.* 7*d.* for 3 years at 3 per cent.

88. Where fractions occur in rate or time it is generally easier to work thus:

Interest on £217 12*s.* 11*d.* for 2 years 7 months at $4\frac{1}{4}$ per cent.

$$\begin{array}{r l}
\text{£} & \\
2\cdot17646 & \\
2\frac{7}{12} & \\
\hline
4\cdot3529 & \\
1\cdot2696 & \\
\hline
5\cdot6225 & \\
4\frac{1}{4} & \\
\hline
22\cdot4900 & \\
1\cdot4056 & \\
\hline
23\cdot896 & = \underline{\text{£}23\ 17s.\ 11d.}
\end{array}$$

(11) Find interest on £308 9*s.* 4*d.* for $3\frac{1}{4}$ years at $2\frac{3}{4}$ per cent.

(12) Find interest on £617 3*s.* 10*d.* for $4\frac{1}{2}$ years at $3\frac{1}{3}$ per cent.

(13) Find interest on £902 13*s.* 7*d.* for $6\frac{2}{3}$ years at $4\frac{1}{4}$ per cent.

(14) Find interest on £1,765 18*s.* 4*d.* for 7 years 2 months at $3\frac{1}{4}$ per cent.

(15) Find interest on £3,186 4*s.* 5*d.* for 3 years 5 months at $4\frac{1}{3}$ per cent.

(16) Find interest on £1,875 16*s.* 11*d.* for 4 years 7 months at $5\frac{1}{4}$ per cent.

(17) Find interest on £818 11*s.* 7*d.* for $4\frac{1}{5}$ years at $3\frac{3}{4}$ per cent.

(18) Find interest on £649 16*s.* 2*d.* for $5\frac{3}{5}$ years at $2\frac{3}{4}$ per cent.

89. In business houses the interest has most frequently to be calculated for an odd number of days. Interest tables calculated to six or seven places of decimals are used, and practice in the use of these tables will show that the interest to the nearest penny can speedily be calculated. A table is here given:

INTEREST AT 5 PER CENT.

£	1 day.	2 days.	3 days.	4 days.	5 days.	6 days.	7 days.	8 days.	9 days.
1	·000137	000274	·000411	·000548	·000685	·000822	·000959	·001096	·001233
2	·000274	000548	·000822	·001096	001370	·001644	·001918	·002192	·002466
3	000411	000822	·001233	·001644	·002055	·002466	·002877	·003288	·003698
4	·000548	001096	001644	·002192	·002740	·003288	·003835	·004383	·004931
5	·000685	·001370	002055	002740	003425	·004109	·004794	·005740	·006164
6	·000822	001644	·002466	·003288	·004109	·004931	·005753	·006575	·007397
7	·000959	·001918	·002877	·003835	·004794	·005753	006712	007670	·008630
8	001096	·002192	·003288	·004383	·005749	·006575	007670	·008767	·009862
9	·001233	·002466	·003698	·004931	·006164	·007397	·008630	·009862	·011100

To calculate by means of the table the intcest on £87 from April 10th to August 17th a t 5per cent. (count in *one* of these days—not both):

No. of days: April, 20 + May, 31 + June, 30 + July, 31 + August, 17 = 129.

Interest on £80 for	9 days	=	·098	62	
,, ,, £80 ,,	20 ,,	=	·219	2	
,, ,, £80 ,,	100 ,,	=	1·096		
,, ,, £7 ,,	9 ,,	=	·008	63	
,, ,, £7 ,,	20 ,,	=	·019	18	
,, ,, £7 ,,	100 ,,	=	·095	9	
			£1·538		= £1 10*s.* 9*d.*

(*Note.*—This table is not sufficient to give accurately interest on very large amounts—tens of thousands of pounds—but it affords practice without being too long.)

If interest is required for odd pence and shillings, an approximation can readily be obtained—the nearest penny is always sufficiently accurate. Thus if amount in example had been £87 16*s*. 10*d*., find first interest on £1 for number of days. Interest on £1 for 129 days = £(·0137 + ·00274 + ·001233).

$$= £\cdot018 = 4d.$$

$$16s.\ 10d. = £\tfrac{5}{6} \text{ approx.} = \tfrac{5}{6} \text{ of } 4d. = 3\cdot3 \text{ pence}$$

$$= 3d. \text{ to nearest penny.}$$

Examples XIII*b*

Interest at 5 per cent.

(1) On £34 for 26 days. (2) On £53 for 25 days.
(3) On £64 for 45 days. (4) On £87 for 63 days.
(5) On £90 for 74 days. (6) On £116 for 85 days.
(7) On £148 for 96 days. (8) On £150 for 100 days.
(9) On £235 for 116 days. (10) On £640 for 148 days.

(11) Find interest on £174 from May 7th to July 6th at 5 per cent.

(12) Find interest on £365 from February 17th, 1919, to May 15th at 5 per cent.

(13) Find interest on £719 from November 10th, 1919, to January 17th, 1920, at 5 per cent.

(14) Find interest on £615 from July 31st to September 29th at 5 per cent.

(15) Find interest on £317 4*s*. 10*d*. from April 19th to August 15th at 5 per cent.

(16) Find interest on £679 13*s*. 5*d*. from June 20th to November 8th at 5 per cent.

(17) Find interest on £185 14*s*. 5*d*. from September 13th to December 17th at 5 per cent.

(18) Find interest on £70 10*s*. 6*d*. from May 27th to August 14th at 5 per cent.

90. Another form of table used is the following. Interest at the rate given at the head of the page is calculated on every £1 up to £100, every £10 up to £1,000, every £100 up to £10,000, etc., from one day to 365 days. This, while making a more cumbersome table, ensures accuracy. The table is, of course, too long to reproduce here, but an extract may be given.

INTEREST AT 6 PER CENT.

	1 day.		2days.		3 days.		5 days.		9 days.		16 days.		18 days.			102 days.			248 days.			365 days.		
	s.	*d.*	*s.*	*d.*	*s.*	*d.*	*s.*	*d.*	*s.*	*d.*	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
1	—		—		—		—		—		0	1	0	0	2	0	0	4	0	0	10	0	1	2
2	—		—		—		—		0	1	0	1	0	0	4	0	0	8	0	1	7	0	2	5
3	—		—		—		—		0	1	0	2	0	0	6	0	1	0	0	2	5	0	3	7
5	—		—		—		0	1	0	1	0	3	0	0	9	0	1	8	0	4	1	0	6	0
10	—		0	1	0	1	0	2	0	4	0	6	0	1	7	0	3	4	0	8	2	0	12	0
40	0	2	0	3	0	5	0	8	1	2	2	1	0	6	4	0	13	5	1	12	7	2	8	0
100	0	4	0	8	1	0	1	8	2	11	5	3	0	15	9	1	13	6	4	1	6	6	0	0
300	1	0	2	0	2	11	4	11	8	11	15	9	2	7	4	5	0	7	12	4	7	18	0	0

EXAMPLE

To find interest on £343 for 102 days at 6 per cent. = interest on £300 + interest on £40 + interest on £3 = £5 0*s*. 7*d*. + 13*s*. 5*d*. + 1*s*. 0*d*. = £5 15*s*.

EXAMPLES XIII*c*

(1) Find interest on £142 for 248 days at 6 per cent.
(2) „ „ £302 „ 48 „ 6 „
(3) „ „ £103 „ 102 „ 6 „
(4) „ „ £455 „ 16 „ 6 „
(5) „ „ £358 „ 48 „ 6 „

CHAPTER XIV

RATES AND TAXES AND BANKRUPTCY

91. RATES are the charges made by a local authority on the occupants of land or buildings within the area controlled by that authority, for the upkeep of its roads, lighting, schools, etc.

Rates are levied upon the annual rental value of the property occupied by the ratepayer, allowance generally being made for repairs, etc.

Taxes are levied by the Government for the upkeep of the Civil Services, the Army, Navy, etc. Taxes are either direct or indirect. Direct taxes are those imposed directly upon the taxpayer; indirect taxes are paid by the consumer of certain commodities on which an import duty has been imposed. The most important direct tax is the Income Tax, the amount of which is fixed yearly, and announced by the Chancellor of the Exchequer in his Budget.

Prior to the war of 1914–18 all salaries of over £160 were taxed at the rate of 1*s*. 2*d*. for every £ in excess of that amount. A married man with children was allowed a further abatement of £10 for each child under 16. The rate has been as low as 2*d*. in the £, in 1874–5. During the war the rate steadily increased, and at the present it stands at 2*s*. 3*d*. in the £ on all incomes over £130. The abatement allowed is £120 on incomes up

to £400, £100 on incomes from £400 to £600, and £70 on incomes from £600 to £700. Besides this a rebate of £40 is allowed for first child, £25 for each other, and £50 for wife if dependent upon her husband. This relates to earned income only. Unearned income, such as that derived from the ownership of property, from investments, etc., bears a tax of 3*s*. in the £.

Example 1.—*A man earns* £350 *a year and investments bring in a further* £40. *He has a wife and two children dependent upon him. What income tax will he be called upon to pay* (*on* 1919 *scale*)?

On earned income the
amount taxable = £350 − £(120 + 115)
= £350 − £235
= £115
£115 at 2*s*. 3*d*. in £ = £12 18*s*. 9*d*.
Unearned income £40 at 3*s*. in £ = £6.
∴ Total tax = £18 18*s*. 9*d*.

Example 2.—*What tax would the man mentioned above have paid at pre-war* (1914) *rate?*

Amount taxable £350 − £(160 + 20) = no rebate for wife
= £350 − 180
= £170
£170 at 1*s*. 2*d*. in £ = £9 18*s*. 4*d*.
£40 unearned at 2*s*. 3*d*. = £4 10*s*.
Total tax = £14 8*s*. 4*d*.

(*Note*.—It is customary for income tax to be deducted from the profits of a company before distribution among shareholders, who therefore receive their dividends "free of tax.")

92. Where a man is not in receipt of a fixed income it is allowable for him to take the average

of the past three years. Should the Income Tax Commissioners suspect that a man is understating his income, the amount of tax imposed is placed at a high figure and the man must then produce documentary evidence (generally an adequate system of bookkeeping) to prove the correctness of his statements.

The yearly life insurance premium may also be deducted on production of the policy or the receipt of the insuring company.

EXAMPLES XIV*a*

(1) A bachelor earning £300 a year insures his life, paying a yearly premium of £8. What will be the tax on his income?

Note.—In this and following sums, current (1919) rates are to be used unless otherwise stated.

(2) A bachelor earns £200 and has property bringing in £150 a year. What is his net yearly income?

(3) A man rents a house at £40 a year, the rates being assessed on £32 per annum. What will house cost him a year if rates are 7*s*. 10*d*. in £ per year?

(4) The owner of a house assessed at £22 10*s*. lets the house to a tenant at 15*s*. per week, the landlord paying rates, which stand at 9*s*. 4*d*. in £ per year, and income tax at 3*s*. in the £. What is his yearly net income from the house?

(5) A man has £500 which he wishes to invest. He may choose between an investment bringing in 4 per cent. free of income tax and one bringing in 6 per cent. on which tax is payable. Which investment will bring in greater net amount and by how much?

(6) The earned income of a married man with two children under 16 is £450. What should be the tax? (Note amount of rebate on incomes over £400.)

(7) A man receives the same salary in 1919 as in 1914, i.e. £350. By how much is the tax greater at the later date if he is a married man with three dependent children?

(8) In 1914 a man earned £350 per annum. He is allowed abatement of £160, a further £10 for each of his two children, and £10 for life insurance. The remainder is taxed at 1*s.* 2*d.* In 1919 his salary has increased to £420. (Note amount of abatement.) By how much has his net salary increased?

(9) A tradesman's profits in 1916 amounted to £315, in 1917 to £368, in 1918 to £415. What tax will he pay in 1919 at 2*s.* 3*d.* in the £?

(10) By how much will a man's net income be increased by a reduction of tax from 2*s.* 3*d.* to 1*s.* 9*d.*, and an increase in the abatement allowed from £120 to £160 if his gross income remains at £300?

(11) A man's net income is £262 when abatement is £120 and tax 2*s.* 3*d.* What is gross income?

(12) By a reduction in the income tax from 2*s.* 3*d.* to 2*s.* and an increase in the amount of abatement from £120 to £160 a man's net income is increased by £7. What is his gross income?

(13) A married man with two children under 16 earns £360 per annum and owns the house he occupies, which is assessed at £42 per annum. How much must he pay in rates and taxes—the former being at 7*s.* 2*d.* in the pound?

(14) The rateable value of a town is £3,765,848. How much will a penny rate produce? The

rates for the half-year are 6*s*. 4*d*. in the £. How much does the town produce per annum ?

(15) I have the choice of purchasing two houses, one assessed at £45 per annum in a town where rates stand at 5*s*. 9*d*. per half-year in £, the other is assessed at £54 and is in a town where rates are 5*s*. 4*d*. per half-year. On which will the rates be the cheaper ?

BANKRUPTCY

93. A man is legally adjudged to be a bankrupt when a declaration has been filed of his inability to meet his liabilities. When a man becomes bankrupt, the Official Receiver takes charge of all his property, which is converted into cash, and the proceeds are divided among the creditors according to the amount owing to them. The amount of cash realised is expressed, in relation to the sum owing, as a "composition" of so much in the pound. The unpaid remainder is called a bad debt, and after paying the composition the bankrupt is no longer legally liable for the rest of the money.

Example 1.—*A bankrupt's assets realise* £795 *and his debts amount to* £1,272. *What composition in the pound can be paid?*

Express $\dfrac{\text{assets}}{\text{liabilities}}$ as the fraction of £1.

$$= \tfrac{795}{1272} = £\tfrac{5}{8} = \underline{12s.\ 6d.}$$

Example 2.—*A bankrupt owes* £726 13*s*. 8*d*. *and his assets realise* £248 10*s*. 5*d*. *How much in the* £ *can be paid?*

$$\frac{\text{assets}}{\text{liabilities}} = \frac{\text{£}248\ 10s.\ 5d.}{726\ 13s.\ 8d.} = \frac{248 \cdot 521}{726 \cdot 683}$$

$72\cancel{6}\cancel{6}\cancel{8}\cancel{3})248521(\cdot 3419 = \text{£}\cdot 342$
218005
30516 $= 6s.\ 10d.$ *to nearest penny.*
29067
.1449

Examples XIV*b*

(1) A bankrupt's assets realise £555. His liabilities amount to £925. How much can he pay in the pound ?

(2) A creditor receives £227 10*s.* as his dividend on a debt of £650. How much is this in the pound ?

(3) A bankrupt pays 7*s.* 9*d.* in the £. How much will a creditor receive to whom £168 6*s.* 8*d.* is owing ?

(4) How much must a creditor to whom is owed £756 by a bankrupt who pays 11*s.* 4*d.* in the £ write off as bad debt ?

(5) Assets realise £336 14*s.* 2*d.* Liabilities = £843 7*s.* 5*d.* How much can be paid in the pound ?

(6) I sell goods which cost me £156 to a firm for £195. The firm fails, paying me 13*s.* 4*d.* in the £ on the £195. What is my real loss ?

Note.—Bad debts are a loss to a firm and may seriously affect the net profits. In order to guard against serious loss from this cause, companies generally reserve a certain amount from their yearly profits (often 5 per cent. of debts owing to them) from which losses by bad debts are met.

Thus a firm may have on December 31st, 1918, a balance from the reserve for Bad Debts Account of £385. During January 1919 the bad debts may equal £32. To this the firm add a further

5 per cent. of their debtors, since it is unlikely that all are financially sound. Suppose the firm has owing to them £8,170, the Profit and Loss Account will show as a loss for the month:

To bad debts	£32	
Add reserve for bad debts	£408 10*s*.	
		£440 10*s*.
Less credit balance of reserve		£385
		£55 10*s*.

This £55 10*s*. is thus set aside by the firm to pay off current month's bad debts and bring up the amount in reserve to the usual 5 per cent. of sundry debtors.

Examples XIV*c*

(1) Sundry debtors (total which purchasing firms still owe to us), £2,140. Bad debts (lost through certain debtors becoming bankrupt), £80. We have a balance of last reserve amounting to £150 and wish to provide 5 per cent. of sundry debtors for bad debts. Show how this is done and what further sum must be added to reserve.

(2) From debtor balances: sundry debtors, £5,770; bad debts, £89. From creditor balances: balance of reserve for bad debts, £150. Reserve 5 per cent. of sundry debtors for bad debts and show amount to be carried to Profit and Loss Account.

The amount of dividend payable to shareholders of a company is decided in much the same way as the amount of a bankrupt's composition.

Example 1.—*If the share capital of a company be* £12,000 *and the profits* £1,500, *the dividend will be* £$\frac{1500}{12000}$ = £$\frac{1}{8}$ *or* 2*s*. 6*d*. *in the* £.

This is generally expressed as a percentage:

$$\tfrac{1}{8} = 12\tfrac{1}{2} \text{ per cent.}$$

Example 2.—*A company's capital is* £125,000 *divided into* £100 *shares. Its profits amount to* £2,342 10*s*. *What will be the dividend per share ?*

Dividend per £100 share $= £\frac{2342 \cdot 5}{1250} = £1\ 8{\cdot}74s.$
$= £1\ 17s.\ 5d.$ per share.

Note.—The dividend declared would probably be $1\frac{3}{4}$ or $1\frac{5}{8}$ per cent.

If the dividend is to be paid free of income tax, the amount at the unearned rate would be deducted.

Dividend would therefore be:
$£2{,}342\frac{1}{2}$ less $(2{,}342\frac{1}{2} \times 3s.)$,
$= £2{,}342\ 10s. - £351\ 7s.\ 6d.$
$= £1{,}991\ 2s.\ 6d.$
Amount per share $£\frac{1991 \cdot 125}{1250} = £1{\cdot}593$
$= £1\ 11s.\ 10d.$
$= 1\frac{3}{5}$ *per cent.* approx.

EXAMPLES XIV*d*

(1) The profits of a company whose capital is £15,000 is £960. What will this be per £ share ?

(2) Capital £7,500, profits £548 10*s*. How much per £ share ?

(3) Capital £25,000, profits £4,765 14*s*. 8*d*. How much per £100 share ?

(4) Capital of a company is £345,000 divided into shares of £100 each. If profits in 1919 amounted to £18,795, how much should a shareholder receive holding 14 such shares ?

(5) An insurance company whose capital is £1,250,000 declared a profit on the year of £194,738. What dividend were they able to pay per hundred pound share free of income tax (3*s*. in £) ?

(6) The gross profits of a company amounted to £64,795. After paying 2 per cent. of profits to managing director and 6 per cent. to holders of preference share capital (£200,000), remainder was divided among ordinary shareholders whose capital amounted to £500,000. What dividend did the ordinary shareholder get on £100 share?

CHAPTER XV

DISCOUNT AND INLAND BILLS OF EXCHANGE

94. No doubt you have often seen in a catalogue of books that some prices are given followed by the word "net" while others are not. It means that retailers are allowed a certain "discount," or deduction from the price of the books not so marked. This is termed *Trade Discount.* This discount varies according to the class of goods, or even with the same goods from time to time.

Let us suppose the Commercial Class Company issues a price list and offers dealers a certain discount, e.g. 25 per cent. or 3*d.* off each shilling. (We may for the purpose adopt any tradesman's catalogue.) Let the class now calculate the cost of a few articles if the trade discount be allowed. Interesting exercises will be provided if such price lists be used as a furniture catalogue, and a room is furnished with and without discount.

E.g. A trade discount of 15 *per cent. is allowed off the list price of furniture. Find cost of a sideboard listed at* £35.

Actual price paid is :

$$100 - 15 = 85 \text{ per cent. of list price.}$$

$$= \frac{\overset{17}{\cancel{85}}}{\underset{\underset{4}{\cancel{20}}}{\cancel{100}}} \text{ of } £\overset{7}{\cancel{35}}.$$

$$= \frac{£119}{4} = £29\ 15s.$$

95. It is the custom in business for the manufacturer or the merchant to give the smaller trader time in which to pay for the goods he buys. The period allowed varies in different trades from one month to six or even more months. As an inducement, however, to pay promptly, a *cash discount* is generally offered in addition to the trade discount. The cash discount is smaller than the trade discount, being generally from $2\frac{1}{2}$ to 5 per cent.

Exercises should be worked from the price lists, such as:

(1) *Goods are marked in the catalogue at £7 10s. net. Payment must be made within a month or $2\frac{1}{2}$ per cent. for cash is given. Find the amount a dealer will save by paying promptly.*

$$2\tfrac{1}{2} \text{ per cent. on £7 10}s. = £\frac{2\frac{1}{2}}{100} \times \frac{7\frac{1}{2}}{1}$$

$$= \frac{\overset{1}{\cancel{5}}}{\underset{\underset{8}{\cancel{40}}}{\cancel{200}}} \times \frac{\overset{3}{\cancel{15}}}{2}$$

$$= £\tfrac{3}{16} = 3s.\ 9d.$$

Or $2\frac{1}{2}$ per cent. $= 6d.$ in every £.

$= 7\frac{1}{2}$ sixpences $= 3s.\ 9d.$

(2) *A trade discount of 30 per cent. is allowed off list price of goods marked £27 10s. and a further $3\frac{3}{4}$ per cent. is given for prompt payment. What should a dealer pay cash for these goods?*

Trade discount $=$ 30 per cent.

$\therefore$ Net price $=$ 70 per cent. of £27 10s.

$= \frac{7}{10}$ of £27 10s.

$=$ £19 5s.

$3\frac{3}{4}$ per cent. $= 9d.$ in £

$= 19\frac{1}{4} \times 9d. = 173\frac{1}{4}d.$

As nothing less than pence are reckoned in calculating discount, call this 173 pence = 14*s*. 5*d*.

∴ Cash price = £19 5*s*. less 14*s*. 5*d*.
= £18 10*s*. 7*d*.

EXAMPLES XV*a*

(1) A trade discount of $33\frac{1}{3}$ per cent. is allowed off prices quoted in a catalogue. Find price to retail dealer of goods marked £7 10*s*., £5 18*s*., £3 16*s*. 6*d*.

(2) A merchant offers a trade discount of 20 per cent. off catalogue prices. Find trade price of goods marked £8 15*s*., £9 17*s*. 6*d*., £24 13*s*. 4*d*.

(3) A manufacturer has goods listed at £33 6*s*. 8*d*. He gives a trade discount of $22\frac{1}{2}$ per cent. What is the trade price of the goods ?

(4) A cycle dealer is allowed 35 per cent. off maker's list prices. What would be the trade prices of cycles listed at £12 10*s*., at £10 10*s*., and at £8 7*s*. 6*d*. ?

(5) A customer buys a cycle at list prices from a dealer who obtains $27\frac{1}{2}$ per cent. trade discount. What would be dealer's total gain if list prices are: cycle, £10 10*s*.; three-speed gear, £2 10*s*.; lamp, 12*s*. 6*d*.; bell, 5*s*.; and carrier, 2*s*. 6*d*.

(6) Deduct $17\frac{1}{2}$ per cent. trade discount and $2\frac{1}{2}$ per cent. cash discount from goods listed at £20.

(7) By paying promptly a dealer obtains 5 per cent. discount in addition to the 20 per cent. trade discount. What would he pay for goods listed at £41 13*s*. 4*d*. ?

(8) A merchant offers 20 per cent. trade discount and 5 per cent. cash discount in addition, off the

list prices. What total percentage of discount does a tradesman obtain who pays cash ?

(9) A tradesman obtains goods listed at £24 at a trade discount of 25 per cent. He adds 25 per cent. on discount price for profit. By how much more or less does the sale price differ from the list price ?

(10) Trade discount is reduced from $33\frac{1}{3}$ per cent. to 25 per cent. By how much must a tradesman raise the price of goods listed in trade catalogues at £36 to maintain a profit of 25 per cent. ?

(11) The lists of Messrs. A. & Co. mark goods at £178 and give a trade discount of 25 per cent. ; the list of B. & Co. mark same goods at £172 and give trade discount of 20 per cent. Which is cheaper, and by how much ?

(12) A manufacturer wishes to gain 20 per cent. on his goods and to offer 25 per cent. trade discount to retailers. At what price must he list goods which cost him 7*s.* 6*d.* each, 18*s.* 4*d.* each, and £1 4*s.* 2*d.* each ?

(13) A manufacturer offers $33\frac{1}{3}$ per cent. off list prices. If he makes 20 per cent. profit by selling goods at trade price, what is actual cost of goods listed at £1, £3 7*s.* 6*d.*, £5 12*s.* 6*d.* ?

(14) Goods marked at £39 bear a trade discount of $33\frac{1}{3}$ per cent., a cash discount of $2\frac{1}{2}$ per cent., while manufacturer's profit is 10 per cent. What is cost to manufacturer ?

(15) Your firm makes 25 per cent. profit on its productions. It gives 5 per cent. discount for cash in addition to trade discount of 20 per cent. By what factor would you multiply the cost of production in order to obtain list price ? Answer to nearest hundredth.

(16) With factor found in number 15 give list prices of goods which cost £2, £3 14*s*., £5 17*s*. 6*d*. to nearest shilling.

(17) Goods are listed at £10 10*s*., and I obtain a trade discount of 25 per cent. I sell the goods at list price. What is my gain per cent.?

(18) Goods listed at 27*s*. 6*d*. per cwt. bear a trade discount of 20 per cent. At what price per lb. should I sell to make a profit of $33\frac{1}{3}$ per cent.? Answer to nearest penny.

(19) My dealer offers trade discount of 20 per cent. and a further $2\frac{1}{2}$ per cent. for cash. I pay cash down for goods listed at £54. At what price can I sell to make a profit of 25 per cent.? (Answer to nearest sixpence.)

(20) Goods are listed at £25 in warehouse (i.e. I must pay carriage, etc.), and a trade discount of 15 per cent. is given. I pay carriage of 12*s*. 6*d*. on goods and sell for £24, giving salesman $1\frac{1}{4}$ per cent. commission. What is my percentage gain to nearest whole number?

(21) A manufacturer reduces the trade discount from 25 per cent. to 15 per cent. By how much per cent. must a dealer raise his price to maintain a profit of 20 per cent.?

(22) The published price of a book is 6*s*., but a retailer gets a discount of $33\frac{1}{3}$ per cent. and thirteen books at price of twelve. What discount can he give (to nearest whole number) off published price and make a profit of 25 per cent.?

(23) A dealer gave 20 per cent. trade discount and a further 5 per cent. for cash. The shortage of material made it necessary for him to give 15 per cent. trade discount with $2\frac{1}{2}$ per cent. for cash. The list price of the goods is £1 12*s*. 6*d*. By how much must I raise my price in order to maintain a

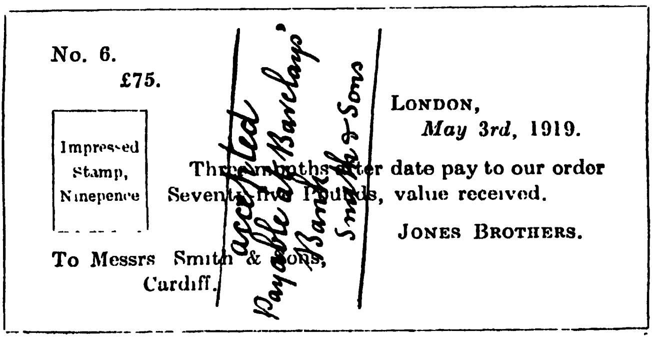
profit of 20 per cent.? (Work to nearest penny throughout.)

(24) Goods cost me 12*s.* 6*d.* to make. At what price must I list the goods to offer a trade discount of 20 per cent. and make a profit of 20 per cent. on trade price?

BANKERS' DISCOUNT AND BILLS OF EXCHANGE

96. The idea has no doubt occurred to you that it may not be always convenient for a merchant to wait for the money for goods sold on credit until payment is due. Documents known as Bills of Exchange make it possible for the merchant to obtain his money when most convenient to him without troubling his debtor before the time has expired.

Let us suppose Jones Brothers have sold goods to Smith & Sons on three months' credit. Jones Brothers draft a bill which they forward to Smith & Sons for their acceptance; that is, for them to acknowledge that they agree to the terms by writing "Accepted" across the bill. The bill is then returned to Jones Brothers like this:


No. 6.
£75.

Impressed Stamp, Ninepence

LONDON,
May 3rd, 1919.

Three months after date pay to our order Seventy-five Pounds, value received.

JONES BROTHERS.

To Messrs Smith & Sons,
Cardiff.

accepted
Payable at Barclays Bank
Smith & Sons

Jones Brothers may obtain cash for this at any time by taking the bill to a banker or bill discounter. Seventy-five pounds less the interest on £75 for the time that must expire before the bill becomes due will be paid for the bill. This interest varies from time to time, but is generally about 3 per cent. Suppose the bill is discounted two months before it is due, the amount charged would be:

$$\frac{75}{100} \times \frac{2}{12} \times 3 = £\frac{3}{8} = 7s.\ 6d.$$

It is customary to allow three days' grace after the bill becomes nominally due, and these three days must be added when calculating the discount. Thus our bill does not really mature until August 6th. The calculation of discount is based upon the number of days the bill has yet to run. Thus if our bill be discounted on June 2nd the number of days would be:

28 in June, 31 in July, 6 in August = 65 days.

The interest would then be:

$$\frac{75}{100} \times \frac{65}{365} \times \frac{3}{1} = \frac{117}{292} = £0{\cdot}401 = 8s. \text{ to nearest } 1d.$$

97. It will be seen that the discount charged by the banker is the interest on the amount of the bill. This is called "Bankers' Discount." True discount, which is little used commercially, is calculated upon the "present value" of the bill, i.e. that amount which when added to the interest on the amount for the given time would produce the amount of the bill. As this method of reckoning discount is of little commercial value, it is omitted from this book.

Interest Tables.—As is mentioned in Chapter XIV, interest tables are commonly used for the calcula-

tion of bankers' discount and interest. From these we find :

Int. on	1 day	5 days	6 days	at 3 per cent.
£	£	£	£	
1	0·00008219	0·00041100	0·00049315	,,
5	0·00041100	0·00205479	0 00246574	,,
7	0·000057534	0·00287680	0·00345204	,,

$$\begin{aligned} \therefore \text{ Int. on } £70 \text{ for } 5 \text{ days} &= 0\cdot0288 \\ \text{,, \quad ,, } £70 \text{ ,, } 60 \text{ ,, } &= 0\cdot3452 \\ \text{,, \quad ,, } £5 \text{ ,, } 5 \text{ ,, } &= 0\cdot0021 \\ \text{,, \quad ,, } £5 \text{ ,, } 60 \text{ ,, } &= 0\cdot0247 \\ \therefore \text{ Int. } £75 \text{ for } 65 \text{ days} &= \underline{\underline{£0\cdot401}} = 8s. \end{aligned}$$

Quickness in the manipulation of tables is a matter of practice, and the following exercises are based upon the table given above, which should be completed by the student.

EXAMPLES XV*b*

(1) Calculate discount on bill for £84 due 36 days hence at 3 per cent.

(2) Calculate discount on bill for £56 due 42 days hence at 3 per cent.

(3) Calculate discount on bill for £98 due 52 days hence at 3 per cent.

(4) Calculate discount on bill for £165 due 28 days hence at 3 per cent.

(5) Calculate discount on bill for £572 due 34 days hence at 3 per cent.

(6) Calculate discount on bill for £1,725 due 66 days hence at 3 per cent.

(7) A bill for £350 is discounted 47 days before maturity. Find discount charged at 3 per cent.

(8) How much will a bill for £275 realise that is

discounted 71 days before maturity, discount being at 3 per cent.?

(9) A bill for £125 due on August 8th is discounted on July 17th. What discount will be charged at 3 per cent.?

(10) This bill was discounted on June 16th. What was received for the bill, if discount is reckoned at 3 per cent.? (Do not forget three days' grace.)

No. 27.
£326.

May 25th, 1919.

Stamp Duty, Four Shillings

Four months after date pay to our order Three hundred and twenty-six Pounds, value received.

POTTER & CO.

Messrs Watts & Co.,
Finsbury.

Accepted
Watts & Co

(*Note.*—Where an odd number of shillings and pence occur in the amount of the bill, the discount on the odd amount may be easily found. The discount on £1 for one year is £$\frac{3}{100}$ = £·03, which is approximately 7*d.* Thus the discount on 16*s.* 4*d.* for 115 days may be taken as equal to discount on 16*s.* 8*d.* (£$\frac{5}{6}$) for $\frac{1}{3}$ year = $\frac{5}{6} \times \frac{1}{3} \times 7d. = \frac{35}{18}$ = 2*d.*)

(11) A bill for £765 18*s.* 4*d.* due on August 14th was discounted on June 10th. What discount would be deducted at 3 per cent.?

(12) A bill for £548 7*s.* 3*d.* due July 10th was discounted on June 6th. Find what the bill realised—discount 3 per cent.

(13) Interest on £1 for 1 day at $2\frac{3}{4}$ per cent. is £0·00007534. Find the interest on £5 and £7 for 1 day, for 4 days, and for 6 days, and hence find discount on bill of £750 for 46 days at $2\frac{3}{4}$ per cent.

(14) From data obtained in No. 13 find the discount charged on bill for £507 for 64 days at $2\frac{3}{4}$ per cent.

(15) Given interest on £1 for 1 day at $2\frac{1}{2}$ per cent. is £·0000685, find the amount realised on a bill for £8,200 drawn on May 27th for four months and discounted on June 20th at $2\frac{1}{2}$ per cent.

(16) From data obtained in No. 15 calculate amount realised on bill for £2,080 due on August 17th and discounted on June 15th at $2\frac{1}{2}$ per cent.

98. Bills of Exchange bear an impressed stamp according to the value of the bill, the rate being:

When amount does not exceed £5, stamp required is 1*d*.

Exceeding:

£5	but not	exceeding	£10,	stamp	required is	2*d*.
£10	,,	,,	£25,	,,	,,	3*d*.
£25	,,	,,	£50,	,,	,,	6*d*.
£50	,,	,,	£75,	,,	,,	9*d*.
£75	,,	,,	£100,	,,	,,	1*s*.

For every additional £100 or part of £100, 1*s*. extra.

99. In business it is usual to keep a record of bills to be paid or honoured by the firm in a "Bills Payable Book"; those for which we are to receive payment are recorded in a "Bills Receivable Book." In these books each bill is numbered and the date when it becomes due is clearly shown, so that in the case of bills payable the firm may be

sure that there is sufficient to its credit in the bank to meet it; it also warns firms to send an "advice note" to the bank authorising them to pay the bill when presented, and with bills receivable to ensure its being sent to the bank for collection in due time.

Should a firm be unable to meet a bill when it falls due, they may ask the holder of the bill to renew the bill for a further period, charging them with the cost of the new stamp and interest for the period of the bill. If a bill is presented at the bank for payment and there be insufficient funds to the credit of the debtor to meet it, the bank "dishonours" the bill. The bill is then handed to a "Notary Public," who formally presents the bill at the bank and "notes" for legal purposes the refusal of the bank to meet it. Noting charges are also added to the amount of the new bill.

A Promissory Note is a promise in writing made by the person owing the money to the person to whom the money is owed to pay the specified sum at a fixed date. A P/N thus differs from a B/E in that the former is drawn by the debtor and needs no acceptance. In book-keeping a P/N is treated in the same way as bills.

100. From *The Times* of May 26th, 1919, we read:

"Money has been in good demand for the greater part of last week, and the rate for day-to-day loans has been firm at 3 per cent., weekly advances commanding $3\frac{1}{4}$ per cent. The banks have restricted their purchases to a very low limit. Mid-June bills have changed hands to-day at $3\frac{7}{16}$ per cent. and the discount houses quoted $3\frac{1}{2}$ per cent. for two-months bills."

To understand this extract we must first realise

that bankers, bill brokers, discount brokers, etc., are really dealers in noney, that the discounting of bills of exchange is, in effect, the loan of money on the security of the bill. When banks, etc., hold more money than they have immediate demand for, they reduce their rates of discount and thus induce holders of bills to discount. Money is then said to be " cheap " and the money market " easy." On the other hand, when there is a good demand for money, or when, for some reason (at present time, the issue of Government stock on advantageous terms), the banks wish to retain their money, the market is said to be " firm." Day-to-day loans are sums borrowed for a single day, but the amounts may be renewed from day to day by agreement. We see, too, that bills having a month to run bear a discount of $3\frac{7}{16}$ per cent., while that on two-months bills is slightly higher.

CHAPTER XVI

FOREIGN BILLS

101. WHEN transactions take place between merchants of different countries, the trouble and risk of transmitting money is avoided by the use of foreign Bills of Exchange. These bills differ somewhat from inland bills, as seen in specimen here given :

> No. 8.
> Exchange for $1,125.
>
> LONDON,
> *March* 28*th*, 1919.
>
> Stamp
> Three
> Shillings.
>
> Sixty days after sight pay this First of Exchange (Second and Third of the same tenor and date not being paid) to the order of James Nelson, the sum of One thousand one hundred and twenty-five dollars, value received, and charge the same to my account.
>
> ARTHUR RUSSELL.
>
> To John R. Rawles,
> 10, First Avenue, New York.

This bill shows that Arthur Russell owes James Nelson of New York the sum of 1,125 dollars for goods. John Rawles of New York owes Russell the same or a greater amount. Russell therefore requests Rawles to pay Nelson the amount named, thereby avoiding the transmission of money. Foreign bills are generally made out in triplicate and the three copies are sent by different routes or at different times, so that should one be lost

another is bound to be received, and the first received cancels the other two.

But suppose Russell owed Nelson of New York the money, and had no debtor in the same town or country to whom he could transfer his debt. It is the work of the bill broker to enable a merchant to meet his liability by payment to another man in London who has money owing to him from New York.

102. This is rendered more difficult not only by the difference in the coinage of various countries, but also by the fact that the relative values of those various coins and the English pound change slightly from time to time. In the countries using gold and silver coinage the same standard of purity and fineness is maintained, so that a certain weight of English gold or silver coins is always worth the same weight of the gold or silver coins of those countries. This fixed relative value is called the *Par of Exchange*. The pars of exchange of the principal countries with which Britain trades are as follows:

				s.	d.
Austria:					
24 kr. 3 hrs.	= £1.	100 hellers = 1 krone	=	0	10
France:					
25 fr. 22 c.	= £1.	100 centimes = 1 franc	=	0	9½
Belgium:					
25 fr. 22 c.	= £1.	100 centimes = 1 franc	=	0	9½
Italy:					
25 fr. 22 c.	= £1.	100 centisimi = 1 fr. or lira	=	0	9½
Switzerland:					
25 fr. 22 c.	= £1.	100 centimes or rappen = 1 fr.	=	0	9½
Greece:					
25 fr. 23 lepta	= £1.	100 lepta = 1 drachme (franc)	=	0	9½
Germany:					
20 marks 23 pf.	= £1.	100 pfennige = 1 reichsmark	=	0	11¾
Canada:					
4·866 dollars	= £1.	100 cents. = 1 dollar	=	4	1
United States:					
4·866 dollars	= £1.	100 cents = 1 dollar	=	4	1

Now, it has been said above that these values change from time to time. Thus we may find that a bill broker may one day reckon £1 as being the equivalent of 25 francs 30 centimes; on another occasion as 25 francs 15 centimes. Various causes contribute to this, but the general effect is that merchants desiring to pay debts in Paris will do so when £1 equals the higher amount rather than the lower, and French merchants will pay debts in London when the lower rate prevails. The bill brokers meet twice a week at the Royal Exchange, London, and settle the current equivalent of £1 in the coinage of other countries, and this list, published on Wednesdays and Fridays in the newspapers, must be consulted by a merchant when buying foreign bills, in order that he may calculate the cost. Owing to the recent war with its disturbance of relative values, the exchange rates differ considerably from the pars of exchange. Thus the rates published on March 28th, 1919, show:

Montreal, \$4.70 = £1	New York, \$4.58 = £1.
Paris, 27.30 fr. = £1.	Italy, 36.50 fr. = £1.

In more normal times (and it is on these we have based our exercises) the prices ran:

Paris, 25.23—25.27 fr.	Naples, 26.95—27.05 fr.
Hamburg, 20.58—20.62 M.	Montreal, 4.85—4.87 dollars.
Antwerp, 25.37½—25.42 fr.	New York, 4.82—4.84 dollars.

From this we learn that a merchant having to *make payments in Paris* would have to pay £1 for every 25.23 francs of his debt, while a merchant

who had *bills payable in Paris* to sell would receive £1 for every 25.27 francs on his bills.

Example 1.—*A merchant owes a dealer in Paris* 5,090 *francs. How much will he pay for a bill on Paris at rate of exchange quoted above? (In buying bill on Paris take the lower figure.)*

Price of bill, 25.23 frs. = £1.

$\therefore$ A bill for 5,090 frs. will cost £$\frac{5090}{25.23}$.

= £201·744 = £201 14*s*. 11*d*.

Example 2.—*Find the cost in New York of a bill for* £248 16*s*. *on London. (In buying bill on London take the higher rate.)*

Cost of bill for £1 = 4·84 dollars.

Cost of bill for £248·8 = 248·8 × 4·84 dollars

= 1,204·19 dollars.

Examples XVI*a*

Use prices quoted above—first column for bills bought abroad, second column for bills bought in London.

(1) What is cost of a bill on Paris for 3,759 francs?

(2) A merchant buys goods in New York to the value of 2,500 dollars. How much will the bills necessary for payment cost him?

(3) An American merchant wishes to pay a bill on London for £570. What will be the cost of the necessary bills?

(4) Find the face value of a bill payable in Hamburg which cost £212 10*s*.

(5) A London merchant owes 798 dollars in Montreal. What must he pay his broker for the necessary bills?

(6) A merchant owed 8,000 dollars to a New York manufacturer. What would be the difference

between the war-time and pre-war costs (as quoted) of the necessary bills ?

(7) What would be the gain to a merchant in London on a debt of 6,375·6 francs by a rise in the rate of exchange from 25.20 fr. to 25.30 fr. ?

(8) A merchant in Italy who owes £900 in London due in March 1919 (see above) arranges to defer payment for six months, paying 4 per cent. per annum for the consideration, in order that the rates of exchange might improve. The rate quoted in September 1919 is £1 = 28.25 francs. How much does he gain or lose by the arrangement ?

103. In the "Money Market" column of the London papers we often see exchanges quoted in following form :

Paris short (or sight) 25.30—25.35.
do. 3 mo. 25.45—25.52$\frac{1}{2}$

From this we know that bills on Paris payable in three months will cost £1 for every 25.45 fr., and that for bills sold in Paris the broker will receive £1 for each 25.52$\frac{1}{2}$ fr. on the bill.

Short, sight, or cheque rates are for bills payable either at sight or within ten days.

The difference in the "short" or "long" rates depends upon the discount rate in the country on which the bill is drawn.

It is sometimes cheaper for the merchant wishing to transmit to Paris to do so at the "long" rate, allowing for the discount. In order to do so he would compare thus :

Suppose rate of discount is 4 per cent. :

4 per cent. per annum = 1 per cent. for 3 months.

$\therefore$ sight rate equivalent to long rate $= \frac{99}{100}$ of 25 fr. 45.
$= 25$ fr. 20 approx.

Since "short" rate £1 buys 25 fr. 30 and "long" rate the equivalent of 25 fr. 20, ∴ short rate is cheaper.

EXAMPLES XVI*b*

Find "short" rate equivalent to following "long" rate, reckoning discount at 4 per cent.:

(1) London on Paris, 3 months, £1 = 25 fr. 40.
(2) London on New York, 3 months, \$1 = 50·2*d*.
(3) London on Amsterdam, 3 months, £1 = 12.55 florins.
(4) London on Naples, 3 months, £1 = 26.70 fr.

Find quotations for three-months bills corresponding to following "short" rates, when rate of discount is 3 per cent.:

(5) London on Paris cheques, 25 fr. 22 c.
(6) London on Amsterdam sight, 12.30 florins.
(7) London on New York sight, 50·8 pence.

(8) A merchant wishes to transmit 2,500 francs to Paris. If £1 = 25.25 fr. and £1 = 12.35 florins, will it be cheaper to pay direct, or through Amsterdam if rate in that city is florin = 2.08 fr.?

(9) How much will it cost a London merchant to send \$2,500 to United States by way of Paris when course of exchange between London and Paris is 25 fr. 60 = £1 Paris and New York 5 fr. 60 c. = \$1?

(10) A Hong Kong merchant wishes to pay a debt of 2,000 rupees. How many dollars must he pay, a rupee being equivalent to 1*s*. $7\frac{7}{8}$*d*., and a dollar (Hong Kong) 4*s*. $1\frac{1}{2}$*d*.?

CHAPTER XVII

STATEMENTS OF ACCOUNT, ACCOUNTS CURRENT

104. FIRMS send to their debtors periodically a statement showing the transactions which have passed between them since the last statement. This is termed a Statement of Account.

STATEMENT OF ACCOUNT

576, WALLBROOK, LONDON,

June 30th, 1919.

Messrs. Codlin & Short,

Dr. to P. Parkins & Co., Ltd.

1919		£	*s.*	*d.*	£	*s.*	*d.*
April 1	To Balance brought forward				38	19	7
24	,, Goods	71	7	6			
May 15	,, ,, ...	49	4	6			
30	,, ,, ...	114	10	0			
June 9	,, ,, ...	53	5	0			
15	,, ,, ...	82	16	6	371	3	6
					410	3	1
April 30	By Cash	100	0	0			
May 18	,, Returns	15	16	6			
June 4	,, Cash	85	0	0	200	16	6
				£	209	6	7

This statement, which is copied from Codlin & Short's account in Parkins's ledger, serves the double purpose of enabling Parkins's account in Codlin & Short's ledger to be checked from it and to remind the latter firm of their indebtedness.

105. The statement is sometimes rendered in the form of an exact copy of the ledger account. It would then appear thus:

LONDON,
June 30th, 1919.

Messrs. Codlin & Short,
In Account with P. Parkins & Co., Ltd.

Dr. *Cr.*

1919		£	*s.*	*d.*	1919		£	*s.*	*d.*
April 1	To Balance b/f .	38	19	7	April 30	By Cash . .	100	0	0
24	„ Goods. .	71	7	6	May 18	„ Returns .	15	16	6
May 13	„ „ . .	49	4	6	June 1	„ Cash . .	85	0	0
30	„ „ . .	114	10	0	30	„ Balance .	209	6	7
June 9	„ „ . .	53	5	0					
15	„ . . .	82	16	6					
	£	410	3	1		£	410	3	1

This form is called an Account Current, or running account.

106. Accounts Current, however, are generally made out with interest charged or allowed upon each item. The following example was sent by an agent who had sold goods on consignment, to the principal. Pending the sale of the goods the principal has obtained part payment in advance by means of bills of exchange (see Chapter XV). Should these bills fall due before June 30th, interest is charged upon them; if after June 30th, interest is allowed; interest is also allowed for the number of days from the date of sale to June 30th.

From this it will be seen that P. Parkins sells goods on behalf of Codlin & Short of Dublin. The latter firm draw on Parkins from time to time and,

ACCOUNT CURRENT

Messrs. Codlin & Short, Dublin,

In Account with P. Parkins & Co., Ltd., London.

Interest at 5 per cent. per annum to June 30th, 1919.

Dr. *Cr.*

		Principal			Days	Interest					Principal			Days	Interest		
1919		£	s.	d.		£	s.	d.	1919		£	s.	d.		£	s.	d.
Jan. 1	To balance b/f . .	150	0	0	181	3	14	4	Feb. 18	By A/S . . .	506	0	0	132	9	3	0
Feb. 25	,, Cash paid to meet								June 12	,, A/S . . .	300	0	0	18	0	14	10
	Sight Draft . .	356	0	0	125	6	1	11	June 30	,, Interest in Red .					1	11	6
Mar. 8	To Acceptance of Draft									,, Balance of Interest	1	1	0				
	due June 8th .	200	0	0	22	0	12	1		,, Balance c/f .	148	19	0				
June 15	To Acceptance of Draft																
	due August 15th .	250	0	0	46	1	11	6									
June 30	To Balance of Interest	.				1	1	0									
	£	956	0	0		11	9	4			956	0	0		11	9	4
July 1	To Balance b/f . .	148	19	0													

as shown by above, they have drawn £150 in excess of sales as shown by A/S rendered. However, there is a balance of interest £1 1*s.* to write off against this.

The interest may be calculated from tables in Chapter XIII.

EXAMPLES XVII*a*

(1) Make out the statement sent to Messrs. Brown & Green by Black, White & Co. of Leeds. On June 1st, 1919, Messrs. Brown & Green buy goods £164; 10th they buy goods £86; 15th they send cheque £200; 25th they buy goods £112; 28th return goods £18, and send cheque £58; statement sent June 30th.

(2) The books of Potter & Dawson of Newport contain the following account:

LAWTON & CO.

Dr. *Cr.*

1919		£	*s.*	*d.*	1919		£	*s.*	*d.*
July 10	To Goods .	37	18	9	July 18	By Cash . .	35	0	0
July 17	,, ,, . .	57	13	0	July 20	,, Returns .	12	18	6
July 24	,, ,, . .	98	16	3	July 29	,, Cash . .	56	0	0

Make out the Statement of Account sent to Lawton & Co. on July 31st.

(3) In ledger of Paul & Pry of Reading is the following account:

READ & SONS

Dr. *Cr.*

1919		£	*s.*	*d.*	1919		£	*s.*	*d.*
May 1	To Balance .	316	15	5	May 15	By Cash . .	169	15	10
May 8	,, Goods .	89	17	3	May 23	,, Bill Receivable . .	310	0	0
May 17	,, ,, . .	145	14	9	May 28	,, Returns .	18	15	0
May 26	,, ,, . .	78	16	8					

Make out the Account Current sent to Read & Sons on May 31st.

(4) Fill in (using table in Chapter XIII) columns 2 and 3 in Account Current.

MESSRS. EVANS & EVANS

In Account with Grey & Co., Leicester

Interest at 5 per cent. per annum on March 31st, 1919

		Principal			Days	Interest					Principal			Days	Interest		
1919		£	s.	d.		£	s.	d.	1919		£	s.	d.		£	s.	d.
Jan. 1	To Balance	110	0	0					Jan. 1	By Cash .	80	0	0				
Jan. 12	„ Goods	86	0	0					Jan. 24	„ Bill at one month .	100	0	0				
Feb. 18	„ „	164	0	0					Mar. 1	„ Cash .	150	0	0				
Mar. 25	„ „	276	0	0													

(5) Make out an Account Current, allowing 5 per cent. per annum interest on each side, from following ledger account in Prim & Proper's books. A/C to be sent on September 30th, 1919.

LONG & CO.

Dr. *Cr.*

1919		£	s.	d.	1919		£	s.	d.
July 1	To Balance .	540	0	0	July 10	By Bill at two months .	500	0	0
July 18	„ Goods .	130	10	0	July 21	„ Returns .	25	0	0
Aug. 5	„ „ . .	381	7	6	Aug. 20	„ Cash . .	200	0	0
Aug. 25	„ „ . .	170	0	0	Aug. 30	„ Sight Draft .	150	0	0
Sept. 3	„ „ . .	212	0	0	Sept. 21	„ Bill at two months .	200	0	0
Sept. 18	„ „ . .	156	2	6					

(6) Complete following A/C:

Messrs. Trew & Hardy,

In Account with Richards & Sons,

Interest at the rate of 5 per cent. per annum to April 30th, 1919.

		Principal			Days	Interest					Principal			Days	Interest		
1919		£	s.	d.		£	s.	d.	1919		£	s.	d.		£	s.	d.
Feb. 1	To Balance b/f . .	160	0	0					Feb. 10	By A/S . . .	240	0	0				
Feb. 25	,, Cash paid to meet Sight Draft . .	112	0	0					Mar. 20	,, A/S . . .	132	0	0				
Mar. 2	To Acceptance due April 2nd . .	210	0	0					Apr. 30	,, Interest in Red .							
Mar. 24	To Cash paid to meet Sight Draft . .	200	0	0						,, Balance of Interest							
Apr. 16	To Acceptance due June 16th . .	175	0	0						,, Balance c/f . .							
Apr. 30	To Balance of Interest																
May 1	,, Balance b/f . .																

CHAPTER XVIII

STOCKS AND SHARES

107. In Chapter V we suggested forming a class into a company for the purpose of affording practice in writing invoices, debit notes, etc. Let us now follow more closely the formation of such a company.

It may be found necessary or advisable, in order to carry on a business needing large works or premises, or in order to exploit a new invention, or to open up a new mine, to raise a larger capital than those directly concerned are able to command. They therefore decide to float a company for the purpose.

The first step is to elect a Board of Directors, which should consist of men with considerable experience of the work which the company is to carry on, and men of unquestioned standing in whom the public may have confidence. This Board of Directors draw up the "Memorandum of Association," which must be registered with the Registrar of Joint Stock Companies, Somerset House, London. This document must show, among other things: (1) the name under which the company is to trade, with the word "Limited" as last word; (2) address of the registered office of the company; (3) the objects of the company; (4) the amount of capital and how it is to be divided. This must be signed by the promoters of the company, who must each state how many shares he intends to

take. If everything is in order, the stamp duties are paid, a certificate of incorporation is issued, and the company is floated.

108. The capital is now called the *Registered, Nominal, or Authorised Capital,* and the directors have no power to issue shares representing a value beyond this registered capital. They may decide that the immediate needs of the business will be met by issuing only a portion of the capital. It is bad business to issue more capital than is necessary, since the dividends must be shared by all holders of issued capital and "over-capitalisation" means money lying idle and smaller dividends. The capital issued is called *Issued Capital,* the difference between that and the Nominal Capital being *Unissued Capital,* which may be issued at such time as the directors may deem necessary.

109. Next must be considered the form of shares into which the Issued Capital will be divided. These may be *Preference Shares,* bearing a fixed rate of dividend which is paid (profits permitting) before any other shareholder receives a portion of the profits; or *Ordinary Shares,* among which is divided the remainder of the profits or a proportion not exceeding a fixed percentage. Sometimes Preference Shares are *cumulative,* i.e. should the profits one year not admit of the percentage dividend being paid, the arrears will be made up as soon as the profits permit. Other forms of shares are *Deferred Shares,* which receive no dividend until the other shareholders have received their fixed dividend, and *Founders' or Management Shares,* which are generally taken by the promoters of the company in payment for their out-of-pocket expenses and work in floating the company.

Should the company need money, they may raise it without increasing the nominal capital by the issue of *Debentures*, which are in reality a loan, generally on some security, e.g. mortgages. These shares bear a fixed rate of interest which *must* be paid whether there be profits or not. In this they differ from the other forms of shares.

110. The company being duly registered, the directors proceed to advertise inviting the public to make applications for shares. This is done by publishing a Prospectus, a copy of which may be seen any day in the newspapers. To the prospectus is generally attached a form which the applicant fills up, stating that he encloses the sum required on application (not less than 5 per cent. of the value of the shares applied for). On the last day of application the directors allot the shares and letters are sent to each applicant—either letters of regret with cheques for return of deposits, or letters of allotment. The names of applicants and amounts sent by them are entered into the "Application and Allotments Book," from which entries are posted into the personal account of each shareholder in the "Shares Ledger." With the letter of allotment the shareholder may receive a further "call," i.e. a demand for further payment on each share. He is also liable for further calls until the shares are fully paid up. The amount of capital actually subscribed is termed the *Paid-up Capital.*

111. Shares are usually issued at *par*, that is, at their exact face value. Occasionally shares are issued at a *premium*, i.e. the cost per share is in excess of their face value, and sometimes at a *discount*, i.e. below face value. In either case the

capital of the company is the nominal value of the shares.

The capital of a company requiring a large initial outlay, as railways, mines, etc., is generally divided into shares of £100 or stock. The chief difference between stocks and shares is that while any portion of stock can be bought or sold, only whole shares can be dealt in.

112. Buying and selling shares or stock is carried on at a special market called the Stock Exchange. This building is not open to the public; a man wishing to buy or sell stock does so through a broker who is a member. On being instructed to purchase certain stock on behalf of an investor, the broker goes to that part of the Exchange at which that particular stock is bought and sold and effects the purchase. He charges a commission or brokerage which ranges from $\frac{1}{8}$ to $\frac{1}{2}$ per cent. (2*s.* 6*d.* to 10*s.*) on every £100 stock. On Government stock brokerage is generally £$\frac{1}{8}$ per cent., on other stocks £$\frac{1}{4}$ to £$\frac{1}{2}$ per cent. The price of stock depends largely upon the demand for it, which in turn depends upon the stability and prosperity of the company. Thus, if a company pays a large dividend, the price of stock will be at a premium; while that paying a small or no dividend will be at a discount. The current prices of stock are published daily in the London papers, together with the rise or fall in the price.

113. A company whose business is flourishing may pay a dividend before the usual time and in addition to the usual distribution. This is termed an *interim dividend.*

In working sums on stocks and shares it is essential to discriminate clearly between the nominal value and the real value of the stock.

Thus, if we read that a man purchased £506 stock for £375, the latter sum is the *real* value, while £506 is the nominal value.

Example.—What income is derived from £3,155 *stock at* 5 *per cent.*?

Interest is paid on the nominal value of the stock, i.e. £5 is received on every £100 stock held.

$\therefore$ Income on £100 stock = £5
„ „ £3,155 „ $= £\frac{3155}{100} \times \frac{5}{1}$
= £157 15*s.*

EXAMPLES XVIII*a*

(1) Find income derived from £2,175 stock at 4 per cent.

(2) What income is derived from £5,280 of 3 per cent. stock ?

(3) What income will be derived from £6,168 10*s.* stock at 5 per cent. ?

(4) Dividend on stock is 6 per cent. What income will a man obtain on £7,958 13*s.* 4*d.* stock ?

(5) A man holds £3,316 15*s.* stock. What will be his income if a dividend of $3\frac{3}{4}$ per cent. is declared ?

(6) Find income derived from £745 15*s.* South Australian $3\frac{1}{2}$ per cent. stock.

114. The income is based on the nominal value of the stock; thus, 4 per cent. interest is £4 on every £100 stock held. Should the price of the stock be at a discount, the real interest will be greater than 4 per cent. as £4 is gained on a less amount of money than £100. In calculating interest where the actual cost of the stock is given, the price must be made the basis.

Example 1.—*What income is derived from investing* £3,894 *in* $3\frac{1}{2}$ *per cent. War Loan at* $89\frac{1}{4}$ *?*

$$\text{Income on } \pounds 89\tfrac{1}{4} \text{ invested} = 3\tfrac{1}{2}$$

$$\text{,, \quad ,, \quad } \pounds 3{,}894 \text{ ,,} = \pounds\frac{3894}{89\frac{1}{4}} \times \frac{7}{2}$$

$$= \pounds\frac{\overset{1298}{\cancel{3894}} \times \overset{2}{\cancel{4}}}{\underset{\underset{17}{\cancel{51}}}{\cancel{357}}} \times \frac{\cancel{7}}{\cancel{2}} = \pounds\frac{2596}{17}$$

$$= \pounds 152{\cdot}706$$

$= £152 14*s*. 1*d*. approx.

Example 2.—*Find income derived by investing* £2,600 *in Japan* 1907 5 *per cent. stock at* 94. (*Allow* $\frac{1}{8}$ *per cent. for brokerage.*)

Note.—The broker in purchasing for the investor charges $\pounds\frac{1}{8}$ for every £100 *stock* he buys, that is, the price per £100 stock becomes $\pounds 94\frac{1}{8}$.

$$\text{Income derived from investing } \pounds 94\tfrac{1}{8} = \pounds 5$$

$$\text{,, \quad ,, \quad ,, \quad } \pounds 2{,}600 = \pounds\tfrac{2600}{94\frac{1}{8}} \times 5$$

$$= \pounds\tfrac{2600\times 8}{753} \times 5$$

$$= \tfrac{104000}{753}$$

$$= 138{\cdot}114$$

$= £138 2*s*. 3*d*. approx.

EXAMPLES XVIII*b*

(1) What income is derived by investing £4,750 in the South African $4\frac{1}{2}$ per cents at 95?

(2) What annual income would be derived from the investment of £2,163 in the Metropolitan Water Board 4 per cents at 63?

(3) I invest £3,218 in the French National

Loan bearing interest at 4 per cent. The current price is £64½. What interest should I obtain ?

(4) The Russian 5 per cents stand at 54. What income should be derived from the investment of £2,368 in this stock ?

(5) What income would be derived from investing £3,635 in North-Eastern Railway stock standing at 93⅝ if dividend paid is 6 per cent. (brokerage ⅛ per cent.) ?

(6) £4,736 is invested in Canadian Pacific Railway at 176½. The C.P.R. is paying 12½ per cent. What income is derived (brokerage ¼ per cent.) ?

(7) What income shall I obtain by investing £696 in £1 shares of the Anglo-Dutch Rubber Co., standing at 41*s.* 6*d.* The dividend per share is 8*s.* 4*d.*

(8) An investor purchases 7 per cent. cumulative preference shares in Premier Oil Co., to value of £6,000, £1 shares standing at 22*s.* 6*d.* What income will be derived ?

(9) In January 1918 a man invests £3,174 in Nitrate stock standing at 114¼ (brokerage ¼ per cent.). He receives an interim dividend of 10 per cent. in June and dividend of 15 per cent. in December. What income did he obtain for the year ?

(10) A man invests £2,000 in 6 per cent. cumulative preference shares standing at 96 in 1915. At the end of year he receives 3½ per cent. only. What income should he have received at end of 1916 ?

115. As has been said, the income is based upon the nominal value of the stock and does not show the real interest on the money invested.

If a man invests money in 3½ Indian stock

standing at $69\frac{5}{8}$ (+ brokerage $\frac{1}{8}$ per cent.), the percentage return on his money will be :

Income derived from £$69\frac{5}{8} + \frac{1}{8}$ is £$3\frac{1}{2}$

" " £100 is $\dfrac{100}{69\frac{3}{4}} \times 3\frac{1}{2}$

$$= \frac{100 \times \overset{2}{\cancel{4}}}{279} \times \frac{7}{\cancel{2}}$$

$$= \frac{1400}{279} = 5{\cdot}02$$

$= £5$ per cent. approx.

EXAMPLES XVIII*c*

What rate of interest is derived by investing in :

(1) 4 per cent. stock at 75 ?
(2) 7 " " " 91 ?
(3) 8 " " " 105 ?
(4) $6\frac{1}{2}$ " " " $94\frac{7}{8}$ (brokerage $\frac{1}{8}$ per cent.)?
(5) 5 " " " $85\frac{1}{4}$ " " "

(6) Which stock offers the better investment, the 7 per cents at 104 or the 4 per cents at 86 ?

(*Note.*—Make imaginary investment of £(104×86). First income = £7 × 86, second income = £4 × 104.)

(7) Which investment is the more remunerative, the 4 per cent. War Loan at £94, or the 5 per cent. at 102 ?

(8) Mexican 6 per cent. bonds stand at 81, the $4\frac{1}{2}$ per cent. Irrigation stock at 59. Which offers the better percentage ?

116. To determine the amount of stock that can be bought for a certain sum of money at given price :

How much stock can be obtained by investing £3,725 in the Great Western Railway, stock standing at 86?

Amount of stock that can be bought for £86	= £100
Amount of stock that can be bought for £3,725	= $\frac{3725}{86} \times 100$
	= £4,331·395
	= £4,331 7*s*. 11*d*. approx.

EXAMPLES XVIII*d*

How much stock can be obtained by investing:

(1) £2,000 in Lancs. & Yorks Railway at $62\frac{1}{2}$?

(2) £3,150 in North-Western Railway at $93\frac{1}{2}$?

(3) £3,000 in Midland Deferred Stock at $56\frac{1}{4}$?

(4) £3,150 in Grand Trunk Debentures at $62\frac{3}{8}$ (brokerage $\frac{1}{4}$)?

(5) £8,630 in Underground Electric at $95\frac{1}{8}$ (brokerage $\frac{1}{4}$)?

(6) What is nominal value of £3,015 invested in Whiteley £1 shares at 22*s*. 6*d*.?

(7) Gramophone Co.'s £1 shares stand at 31*s*. What will be nominal value of £258 17*s*. invested in them?

(8) £4,208 is invested in British Borneo Oil £1 shares at 26*s*. 3*d*. What is nominal value of shares (brokerage $\frac{1}{4}$ per cent.)? (*Note*.—Price of £100 shares = (£100 × 26*s*. 3*d*.) + 5*s*.).

(9) Indian Bank stock stands at $75\frac{1}{2}$. What amount of stock can I purchase for £3,850 (brokerage $\frac{1}{8}$)?

(10) B.S.A. £1 shares stand at 35*s*. 3*d*. What can my broker obtain for me for £1,014 17*s*. 6*d*., his commission being $\frac{1}{4}$ per cent.?

(11) What will be the cost of £508 Indian stock at 74 ?

(12) I purchase £3,000 Grand Trunk stock at 63. What must I pay for it ?

(13) The price of Burmah stock is $65\frac{5}{8}$. What is value of £1,365 stock (brokerage $\frac{1}{8}$) ? (*Note.*—Price realised for £100 stock is £$65\frac{5}{8} - \frac{1}{8}$.)

(14) I hold £3,000 Armstrong £1 shares. What is their present value at 37*s.* per share ?

(15) A man sells £2,135 £5 shares standing at £3 2*s.* 6*d.* per share. What does he realise ?

(16) Maypole Dairy Deferred £1 shares stand at 21*s.* How much should I pay for 1,200 such shares ? What per cent. shall I get on my money if dividend per share is 3*s.* 5*d.* ?

SUNDRY NOTES ON STOCKS

117. A *Contract Note* is the note sent by the broker to the investor advising him of the sale or purchase of stock on his behalf. A contract note for purchase or sale of stocks must bear a shilling stamp if for more than £100 and less than £500 and 1*s.* for every £500. A contract note takes the following form :

LONDON,
June 30*th*, 1919.

We have to-day purchased to your order, subject to the Rules, Regulations and Customs of the London Stock Exchange :

		£	*s.*	*d.*
£1,000 Great Western stock at	97	970	0	0
Brokerage . . .	$\frac{1}{2}$%	5	0	0
		£975	0	0

In actual business the brokers frequently charge for Government stocks $\frac{1}{8}$ to $\frac{1}{4}$ per cent. on the nominal par value of the stock. This charge (2*s.* 6*d.* or 5*s.*) is charged also on a part of £100 stock.

118. *Consols* is a contraction for "Consolidated Funds" or "Consolidated Annuities." These funds or annuities are the amalgamated debts of the country contracted at various times. Consols bear an interest of $2\frac{1}{2}$ per cent. and are preferred by those investors who prefer absolute safety of their capital to a large or fluctuating income. During the years 1914–1919 the Government raised funds by means of "war loans"—subscribers loaning their money to the Government for the purpose of carrying on the war. These loans bore interest of from $3\frac{1}{2}$ to 5 per cent., and as a consequence people sold out their Consols to invest in the "loans," so that in June 1919 Consols had fallen to $52\frac{7}{8}$.

119. *The "Victory Loan."*—In June 1919 the Government issued two new loans termed the Four Per Cent. Victory Bonds and the Four Per Cent. Funding Loan. The distinctive features of these loans are:

I. Four Per Cent. Victory Bonds

(1) Issued in bonds of from £5 to £5,000 (nominal value).

(2) Price of issue is 85 per cent., which may be fully paid on application or in instalments extending over six months.

(3) Interest at rate of 4 per cent. per annum to be paid on March 1st and September 1st annually. (To meet this the Government under-

take to set aside each half-year $2\frac{1}{4}$ per cent. of nominal amount of bonds issued. After deducting anount required for payment of interest for the half-year, the balance will be carried to a Sinking Fund which will be applied by means of annual drawings to the redemption of bonds at par.)

(4) The amount of the issue is unlimited.

(5) The interest is to be exempt from income tax.

(6) Stock and bonds of certain former issues of War Loan and Exchequer Bonds will be accepted at par as the equivalent of cash.

(7) Principal and interest of loan will be a charge on Consolidated Funds (Consols).

II. Four Per Cent. Funding Loan

(1) Issued in stock or bonds in multiples of £50.

(2) Price of issue is 80 per cent.—either fully paid on application or in instalments extending over six months.

(3) Loan will be redeemed at par between 1960 and 1990—from 41 to 71 years after issue.

(4) Dividends payable May 1st and November 1st each year at rate of 4 per cent. per annum.

(5) Issue is unlimited.

(6) Interest will be free of income tax.

(7) Stocks and bonds of certain former issues of War Loan and Exchequer Bonds will be accepted at par as the equivalent of cash.

(8) Principal and interest to be a charge on Consolidated Funds.

Examples based on "Victory Loan"—XVIII*e*

(1) A man invests money in 4 per cent. Victory Bonds at 85. What is the real rate of interest ?

(2) What is real rate of interest obtained by investing in 4 per cent. Funding Loan at 80 ?

(3) A man invests £2,125 in Victory Bonds at 85 ; what income will he derive from his investment ?

(4) The interest paid on Victory Bonds at 85 is £4, which is free from taxation (3*s.* in £). What per cent. does this represent ?

(5) Interest on Funding Loan is free of tax. What is real per cent. of interest ?

(6) A man purchasing a Victory Bond at £85 has it bought back at par after receiving two half-yearly dividends. What profit does he make on his outlay ? What per cent. profit is this, adding exemption from income tax on dividend ? Answer to nearest tenth.

(7) A man buys £750 $4\frac{1}{2}$ per cent. (1925–1945) War Loan at 99. How much does he pay ? He converts this into Victory Bonds, £100 loan being accepted as £100 cash. What is nominal value of bonds that he obtains ?

(8) Two men each invest £1,360, one in the Victory Bonds at 85, the other in Funding Loan. After two years the man investing in Victory Bonds has two of his bonds redeemed at par. Which man has obtained the greater interest, and by how much ? (Ignore income tax.)

(9) In 1918 a man invested £1,425 in 5 per cent. Exchequer Bonds at 95. On the issue of the new loan he converted the bonds into 4 per cent. Funding Loan at 80 each, Exchequer Bonds being accepted as equivalent of £100 cash. How much does he gain or lose in income by the change ?

(10) A man has £4,785 New Zealand $3\frac{1}{2}$ per cent. stock, on the interest of which he pays income tax at 3*s.* in £. He sells at 78 and invests

proceeds in 4 per cent. Funding Loan at 80, interest free of income tax. What does he gain per annum by the change?

120. *Combines and Trusts.*—There is a tendency at present for large firms in the same business to combine their capitals and trade as one firm. The combination of capitalists forming the Standard Oil Trust in America in 1882 resulted in that company obtaining a virtual monopoly of the oil trade and squeezing out the small producer. While economies by producing on a large scale, by better organisation, and the prevention of overlapping are undoubtedly effected, the question as to whether these combinations will result in benefit to the consumer is more than doubtful and presents one of the most serious of the present economic problems.

CHAPTER XIX

CALCULATION OF COSTS, FREIGHT, INSURANCE

121. WE are all familiar with the terms "post free" or "carriage paid" associated with the price of goods in advertisements. It is clear that the seller has allowed for the cost of carriage in quoting his price. There are other methods of quoting price, each method having its recognised phrase as follow :

F.A.S.—Free alongside ship—means that the seller takes responsibility to the ship side, from which time all charges and responsibility fall upon the buyer.

F.O.B.—Free on board—means that seller takes responsibility and bears cost until goods are put on board.

F.O.R.—Free on rail—means that seller pays all charges until goods are on the train.

"Loco"—denotes that price quoted is for the price of goods where they lie, and includes no charge for removal.

C. & F.—cost and freight—price here includes all charges to the port of destination ; similar to "carriage paid" with inland goods. C. & F., however, does not include insurance nor import duties.

C.I.F.—cost, insurance and freight—as term indicates, price quoted include C. & F. plus insurance of goods.

Free, Franco, or Rendus—means that cost includes *all* charges, import duties, carriage to warehouse of purchaser, etc.

122. In "costing" an article, particularly a manufactured article, the expenses incurred in

material and labour must be carefully recorded in order that such a price may be put upon it that will give the customary percentage of profit and yet not exceed the price of similar articles shown by competitors.

In manufacturers' offices, therefore, accurate "cost accounts" are kept showing the cost of each process in the manufacture of the article.

The cost of a piece of work includes (*a*) cost of raw material, (*b*) cost of labour, (*c*) establishment expenses, as cost of motive power, foremen, rent, taxes, depreciation of machinery, etc., (*d*) indirect charges (sometimes called on-cost), as salaries of salesmen and clerks, travellers' commissions, advertising, interest on capital. To this must be added cost of carriage if goods are delivered "carriage paid," insurance if c.i.f., etc.

Many firms have printed sheets or cards recording each item of raw material, each process of manufacture with ruled columns for quantity produced, and the cost of each item.

Example :

(1) *Upholstered chair* .

	£	*s.*	*d.*
Frame	0	8	6
Castors, set	0	1	6
Castor rings	0	0	6
Webs, 12 yds. @ 2½*d.*	0	2	6
Scrim, 1½ yds. @ 1*s.* 4*d.*	0	2	0
Hessian, 1½ yds. @ 1*s.* 4*d.*	0	2	0
Springs, 12 seat @ 3*d.*	0	3	0
Springs, 8 arm @ 2*d.*	0	1	4
Tacks and twine	0	0	6
Stuffing—hair, 12 lbs. @ 2*s.*	1	4	0
Stuffing—wadding, 2 lbs. @ 2*s.* 6*d.*	0	5	0
Gimp, 8 yds. @ 7*d.*	0	4	8
Cord, 4 yds. @ 5½*d.*	0	1	10
Calico, 1½ yds. @ 1*s.* 6*d.*	0	2	3
Damask or tapestry, 3¾ yds. @ 10*s.*	1	17	6
Total cost of material	£3	15	3

Labour

	£	s.	d.
Upholsterer, 24 hrs. @ 1s. 6d. . . .	1	10	0
Woman (sewing), 4 hrs. @ 8d. . . .	0	2	8
Cabinet maker (fixing castors, legs, etc.) 1 hr. @ 1s. 6d.	0	1	6
	£1	14	2

Total cost under (*a*) and (*b*), £5 9*s*. 5*d*.

Allowing 5 per cent. for establishment charges and 10 per cent. for "on-cost," we get £5 9*s*. 5*d*. + 5*s*. 6*d*. + 11*s*. = £6 5*s*. 11*d*. Add to this 25 per cent. profit = £1 11*s*. 5*d*. = £7 17*s*. 4*d*. Thus the chair may be priced £7 17*s*. 6*d*.

(2) *Printing small books*—1,000 *copies:*

Materials

	£	s.	d.
Paper . . .	7	10	0
Ink . . .	1	18	0
Boards . .	3	2	6
Leather . .	3	0	0
Cloth . . .	5	8	0
Sundries . .	0	7	6
Total . .	£21	6	0

Machines

	£	s.	d.
Setting up . .	0	12	0
Machinery . .	1	10	0
Washing . .	0	3	0
Total . .	£2	5	0

Printers

	£	s.	d.
Composition	4	4	0
Distribution . .	1	2	0
Imposition . .	0	10	0
Reading . .	0	5	0
	£6	1	0

Binders

	£	s.	d.
Ruling . . .	2	6	0
Cutting . .	0	12	6
Folding and sewing	1	5	0
Binding . . .	1	6	0
Finishing . .	0	18	0
Sundries . .	0	14	0
Total . .	£7	1	6

Total Cost

	£	s.	d.
Materials	21	6	0
Printers	6	1	0
Machines	2	5	0
Binders	7	1	6
	£36	13	6

	£	s.	d.
Allow establishment expenses 10 per cent.	3	13	4
On-cost 5 per cent.	1	16	8
Total	42	3	6
Profit 20 per cent.	8	8	8
Total cost for 1,000 copies	£50	12	2

Approximate cost, 1*s.* 1*d.* per copy.

Examples

(1) A manufacturer sends goods to agent to sell on commission. At what must he price them if raw material costs £16 7*s.* 6*d.*, labour £24 6*s.*, establishment and on-cost £3 4*s.*, cost of carriage 12*s.*6*d.*? Agent is to obtain $2\frac{1}{2}$ per cent. on sales, and manufacturer wishes to clear 10 per cent. (*Note.*—The 10 per cent. is to be made on cost price, thus add 10 per cent. to total cost—this represents $97\frac{1}{2}$ per cent. of selling price.)

(2) I wish to price goods carriage paid in United Kingdom. The raw material costs £2 15*s.*, labour £1 8*s.*, establishment and on-cost, 15*s.* The average cost of carriage is 4*s.* 6*d.* Profit, $12\frac{1}{2}$ per cent. of total costs (exclusive of carriage). What should I charge?

(3) I wish to tender for the supply of dinners to a large school. I estimate following as average material required daily: 1 cwt. meat at 1*s.* 3*d.* per lb.; 2 cwt. potatoes at 8*s.* per cwt.; 1 cwt. flour at 19*s.* 6*d.* per cwt.; vegetables, £1; fruit or jam, 24*s*; labour: cooks, three for four hours at 1*s.* 3*d.* per hour; carvers and servers, 10 at 3*s.* 6*d.* per day; cleaners, 4 for two hours at 8*d.* per hour. At what price per head could I serve the 400 dinners and make a profit of 25 per cent. (nearest penny)?

(4) A merchant purchases 1,000 cases tinned fruit at 16*s*. per case f.o.b. Halifax (Nova Scotia). Carriage charged is £8 10*s*.; insurance, 2*s*. 6*d*. per cent.; dock dues and carriage to warehouse, £2 5*s*. If each case contained 24 tins, at what rate per dozen can he sell the tins to clear 20 per cent. and allow average cost of 2*s*. 6*d*. per doz. for delivery ?

(5) A brickmaker's cost sheet shows following for production of 50,000 bricks :

Labour	£	*Materials*	£
Digging clay	315	Coal and coke	294
Kilning	410	Sundries	86
Making, etc.	363		
Stacking, carting, etc.	218		

Establishment expenses at 10 per cent., on-cost at $2\frac{1}{2}$ per cent. of prime costs. At what price per thousand can he sell the bricks to clear 15 per cent. ?

(6) A buyer buys linoleum at 1*s*. 10*d*. per yard less trade discount of 10 per cent. He pays men 3*d*. per yard for laying, and wishes to clear 25 per cent. profit. At what price must he mark the linoleum per yard, laying free ?

(7) A man estimates for redecoration of a building. The materials required would be 40 lbs. paint at $7\frac{1}{2}d.$ per lb., 20 lbs. enamel at 2*s*. 3*d*. per lb., 24 rolls paper at 8*s*. 4*d*. per roll. Sundries, 15*s*. 9*d*. Labour, four men, 48 hours each at 1*s*. 3*d*. per hour; boy, 48 hours at 4*d*. per hour. Use of tools and plant, 15*s*.; insurance of men under Compensation Act, 4*s*. Allowing profit of 15 per cent., what would be amount of tender ?

CHAPTER XX

BANKING

123. We have already learnt that one very important part of the work of a bank is discounting bills of exchange. But we also know that a bank is a safe repository for money which may be withdrawn on demand by means of cheques.

When opening an account at a bank we may either place our money "on deposit" or "on current account." Deposit account is used for such large sums of money that will not be required by the depositor at short notice, and earns an interest for him which increases slightly with the greater length of notice he is prepared to give on withdrawing. The deposits on which we may wish to draw at any time without giving the bank preliminary notice are placed on current account. In most London banks it is customary to require a customer to maintain a minimum balance of £50 on current accounts, on which no interest is paid and no commission charged. In some banks and in provincial banks 2 per cent. is allowed on credit balances, while 2*s.* 6*d.* per cent. commission is charged on the total amount of cheques drawn. When paying in money to the bank the customer makes out a credit or paying-in slip—both the perforated slip and the counterfoil. A specimen paying-in slip is here shown.

The Northern Bank, Ltd.
June 29th, 1919.

CREDIT

The Northern Bank Ltd.

H. Edwards.

in A/C with

June 29th, 1919.

	£	s.	d.	Cheques on other Banks. £	s.	d.	Cheques on Ourselves. £	s.	d.
Bank of England Notes .	100	0	0	44	8	5	25	0	0
Country Notes . .	20	0	0	12	9	10	10	10	0
Gold { Sovs. . . .	45	0	0	33	10	0	10	0	0
Gold { Half Sovs. . .	12	0	0	21	17	3			
Silver	8	10	0						
Total of Cheques . .	157	15	6						
Total of Bills . .	268	18	4						
Total . .	£612	3	10	£112	5	6	£45	10	0

	£	s.	d.
Bank of England Notes . .	100	0	0
Country Notes	20	0	0
Gold { Sovereigns . . .	45	0	0
Gold { Half Sovereigns . .	12	0	0
Silver, etc.	8	10	0
Total Cash . . .	185	10	0
Total of Cheques on Other Banks .	112	5	6
Total of Cheques on Ourselves .	45	10	0
Total Bills as per Back . .	268	18	4
	£ 612	3	10

From *H. Edwards*

Signature of Party paying in *H. Edwards.*

The slip is torn from the book after the amounts have been checked, and from these credit slips the ledger clerk credits the account of the depositor. The counterfoil is initialled by the cashier and is a receipt.

Withdrawals are made by means of cheques which are issued in books to the depositor. Each cheque bears an impressed twopenny stamp, which is a tax upon the depositor and goes to the State. A cheque, after it is paid, becomes a debit slip, and from it the ledger clerk debits the customer's account. An account in the ledger of a bank is kept in the following manner:

J. H. Green *James Henry Green*

Date.	Debtor.	Creditor.	Debtor.	Creditor.	Balance.	Days.	Interest.
			£ s. d.	£ s. d.	£ s. d.		s. d.
1919 June 1		J. Green		150 0 0	150 0 0		
		P. Murray		32 10 0	182 10 0	9	1 9
,, 10	Self		20 0 0		162 10 0		
	H. Porter		12 10 0		150 0 0	2	0 4
,, 12		Cash		15 0 0	165 0 0		
		S. Samuels		64 0 0	229 0 0	8	2 0
,, 20	R. Roberts		27 15 0		201 5 0	5	1 1
,, 25		J. Storer		28 15 0	230 0 0		

124. It will be seen from above that cheques drawn by the depositor, J. H. Green, for himself H. Porter, and R. Roberts, are copied in the debit columns, from the credit slips the creditor columns are copied. The balance column shows the balance to the credit of the depositor at any time, while the interest is allowed at the rate per cent. (here 2), and is carried forward in interest column. The amount of interest is found from interest tables. The signature, J. H. Green, above the account,

is a pasted-on slip signed by Green and used for identification of his signature upon a cheque. Banks make up their books twice a year, and add accumulated interest and deduct commission.

125. Another method of calculating the interest is by multiplying the balance by the number of days thus: $(182\frac{1}{2} \times 9) + (150 \times 2) + (229 \times 8) + (201\frac{1}{4} \times 5) = 1{,}642\frac{1}{2} + 300 + 1{,}832 + 1{,}006\frac{1}{4} = 4{,}780\frac{3}{4}$.

The interest is then found on £4,780 for 1 day at 2 per cent. = £·258 = 5*s*. 2*d*. This, too, is calculated and added on each half-year.

The third method is to calculate the interest once per month on the minimum monthly balance —in example this would be £150. Interest would therefore be that on £150 for 1 month at 2 per cent. $= £\frac{3}{2} \times \frac{2}{12} = 5s.$

126. It will be seen by this method that if a man's balance is greatly reduced at any part of the month the interest suffers. For that reason it is common for firms to make monthly payments on the last day, so that the cheques will not be debited against them until the beginning of the following month.

Examples XX

For the purposes of obtaining practice, use 5 per cent. table in Chapter XIII.

(1) Complete the following ledger account:

(*a*) using direct calculation of interest.
(*b*) using method of multiplication.
(*c*) using minimum monthly balance.

THOMAS MARVIN

Date.	Debtor.	Creditor.	Debtor.	Creditor.	Balance.	Days.	Interest.
1919			£ s. d.	£ s. d.	£ s. d.		£ s. d.
June 1		H. Jarvis		16 15 0			
		T. Grover		35 18 6			
		Cash		45 17 8			
„ 9	Self		15 0 0				
	P. Huson		10 0 0				
„ 16		H. Harper		39 15 4			
		R. Riches		17 17 6			
		Cash		18 0 0			
„ 19	S. Stalker		39 16 7				
		P. Partridge		56 15 5			
„ 26	R. Rouse		18 16 8				
	E. Linney		15 15 0				
„ 29		R. Done		37 12 10			
„ 30		Interest (5%)					
	Commission on cheques drawn @ 2*s.* 6*d.* per cent.						
		Balance					

(2) Make out H. Hall's current account. On July 1, 1919, he opened an account by paying in a cheque drawn by R. Bullock, £200, and cash £50; 3rd, drew cheque for self, £20; 5th, paid in cash £18 10*s.* and cheque (W. Bradley) £68; 9th, drew cheques for B. Wheeldon, £29, and E. Jolley, £54; 15th, sent cheque to H. Hatter, £39 10*s.*; 21st, paid in cheques from J. Murphy, £30, and from P. Hardy, £47 10*s.*; 26th, drew cheque for self, £35. What is balance of interest for month? What is interest on minimum monthly balance?

(3) Make out a paying-in slip for H. Cole, who on July 22, 1919, paid into the Royal Bank the following: Two £10 and six £5 notes, £58 in £1 Treasury notes, and £32 in 10*s.* Treasury notes (regard as gold); silver and copper, £44 10*s.*; cheques—following on Royal Bank, £17 17*s.* 6*d.*, £23 15*s.* 8*d.*, £41 17*s.* 4*d.*, and on other banks £59 15*s.*, £72 14*s.*, and bill of exchange £75.

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(4) Make out a current account for P. Greenwood:

1919			£	s.	d.	1919			£	s.	d.
May	1.	Cr. .	385	0	0	June	1.	Cr. .	214	16	5
,,	10.	Cr. .	19	19	6	,,	6.	Dr. .	32	18	6
		Cr. .	27	14	0	,,	9.	Cr. .	66	16	6
,,	19.	Dr. .	163	15	0	,,	10.	Cr. .	38	18	8
,,	26.	Dr. .	32	13	4	,,	27.	Dr. .	25	0	0
,,	30.	Cr. .	59	19	0	,,	29.	Dr. .	16	10	0

Find and credit balance of interest.

Find interest on minimum monthly balance for May—credit interest and carry balance forward to June. Find also minimum monthly balance and interest thereon for June.

CHAPTER XXI

SYMBOLICAL EXPRESSION

127. In illustrating certain arithmetical processes it is often found convenient to substitute letters instead of numbers, the advantage being that each letter can be looked upon as indicating any number whatever, so that the processes are thereby made perfectly general and applicable to any numbers which may occur. As the method is dependent to some extent upon a knowledge of the elementary rules of algebra, a brief record of these and some of their applications to arithmetic are given below.

128. Let $a\ b\ c\ \ldots$ be letters representing certain numerical quantities, so that, when in future we refer to any particular letter, it is to be understood that this is simply an abbreviated manner of referring to the numerical quantity represented by this letter.

An *expression* is a collection of numbers or letters which are connected by any of the four arithmetical signs $+$ $-$ $\div$ or $\times$. The parts of an expression separated by the sign $+$ or $-$ are called *terms*.

129. When letters are multiplied together the multiplication sign may be omitted, e.g.:

$$a \times b \times c \text{ may be written } abc.$$
$$8 \times x \times y \text{ may be written } 8xy.$$

It is obvious that two figures could not be written

side by side in the same manner, e.g. 34 indicates $30 + 4$, not 3×4.

130. If a number or letter is repeated several times as a factor it is said to be raised to a given power (para. 8). Thus $a \times a \times a$ is written a^3 and is termed the *third power* of a; $b \times b \times b \times b$ is written b^4 and is termed the *fourth power* of b.

Powers of like letters are multiplied together by adding their indices, e.g.:

$$a^3 \times a^4 = a \times a \times a \times a \times a \times a \times a = a^{3+4} = a^7$$
$$b^5 \times b^3 = b^{5+3} = b^8$$

and in general, if m and n are any numbers whatever, $a^m \times a^n = a^{m+n}$.

Examples:

$$\begin{aligned}
\text{If } a &= 4, b = 5, c = 6, m = 3, n = 2, \\
\text{then } 8ab &= 8 \times 4 \times 5 = 160; \\
3a^3b^2 &= 3 \times 4 \times 4 \times 4 \times 5 \times 5 = 4{,}800; \\
5a^m &= 5 \times 4^3 = 5 \times 4 \times 4 \times 4 = 320; \\
2a^nb + 3c^m &= (2 \times 4^2 \times 5) + (3 \times 6^3) \\
&= 160 + 648 = 808; \\
4ab \times 3a^2b^3 &= 12a^3b^4 = 12 \times 4^3 \times 5^4 \\
&= 12 \times 64 \times 625 = 480{,}000.
\end{aligned}$$

131. When an expression is to be raised to a given power it should be enclosed within a bracket, and the index of the power written outside the bracket, e.g.:

$2a + 3b - 5c$ raised to the third power is written $(2a + 3b - 5c)^3$.

$5a^2$ raised to the fourth power is written $(5a^2)^4$.

The latter example can easily be developed further; thus:

$(5a^2)^4 = 5a^2 \times 5a^2 \times 5a^2 \times 5a^2 = 625a^{2+2+2+2} = 625a^8$.

Again $(b^3)^5 = b^3 \times b^3 \times b^3 \times b^3 \times b^3 = b^{3 \times 5} = b^{15}$ or generally $(x^m)^n = x^m \times x^m \times \ldots n$ times $= x^{m + m \ldots n \text{ times}} = x^{mn}$.

Thus the power of a power of an expression can be obtained by multiplying together the two indices.

Example 1.—*If* $a = 2$, $b = 6$, $c = 3$. *Find the value of* $(2a + 3b - 5c)^3$.

Result $= (4 + 18 - 15)^3 = 7^3 = 343$.

Example 2.—*Find the value of* $(x^m)^n$ *if* $x = 4$, $m = 2$, $n = 3$,

$(x^m)^n = x^{mn} = 4^6 = 4096$.

132. When several terms are to be multiplied by the same factor, they can be enclosed within a bracket and the factor written outside, e.g.:

$3a + 3b + 3c = 3(a + b + c)$.

$ma + mb + mc + md = m(a + b + c + d)$.

$lh + bh + lh + bh = 2lh + 2bh = 2h(l + b)$.

Example 1.—*The area of the walls of a rectangular room can be represented by the formula* $A = 2h(l + b)$. *Find the area if* $l = 15$ *ft.*, $b = 12$ *ft.*, $h = 10$ *ft.*

Area $= 20(15 + 12) = 20 \times 27 = 540$ sq. ft.

Example 2.—*Find the value of* $ma + mb + mc$ *if* $m = 12$, $a = 12{\cdot}7$, $b = 13{\cdot}4$, $c = 13{\cdot}9$.

$$\begin{aligned} ma + mb + mc &= m(a + b + c) \\ &= 12(12{\cdot}7 + 13{\cdot}4 + 13{\cdot}9) \\ &= 12 \times 40 = 480 \end{aligned}$$

133. The following results have important arithmetical applications:

$$\begin{aligned} (1)\ (a + b)^2 &= (a + b)(a + b) \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2 \end{aligned}$$

$$\begin{aligned}(2)\ (a-b)^2 &= (a-b)(a-b)\\ &= a^2-2ab+b^2\end{aligned}$$

$$\begin{aligned}(3)\ (a+b)(a-b) &= a(a-b)+b(a-b)\\ &= a^2-ab+ab-b^2\\ &= a^2-b^2\end{aligned}$$

Examples :

$$\begin{aligned}\text{(i)}\quad 109^2 &= (100+9)^2\\ &= 100^2+(2\times 9\times 100)+9^2\\ &= 10{,}000+1{,}800+81\\ &= 11{,}881\end{aligned}$$

$$\begin{aligned}\text{(ii)}\quad 998^2 &= (1{,}000-2)^2\\ &= 1{,}000^2-(2\times 2\times 1{,}000)+2^2\\ &= 1{,}000{,}000-4{,}000+4\\ &= 996{,}004\end{aligned}$$

$$\begin{aligned}\text{(iii)}\quad 82\times 78 &= (80+2)(80-2)\\ &= 80^2-2^2\\ &= 6{,}400-4\\ &= 6{,}396\end{aligned}$$

$$\begin{aligned}\text{(iv)}\quad 54^2-36^2 &= (54+36)(54-36)\\ &= 90\times 18\\ &= 1{,}520\end{aligned}$$

134. The area of any regular rectangular border can be calculated in the following manner :

(*a*) Let the border be outside the rectangle, of which the length $= l$, and breadth $= b$. Let width of border be d.

Then outer dimensions of border are $(l+2d)$ and $(b+2d)$ respectively.

Area of the whole $= (l+2d)(b+2d)$.

Area of inner rectangle $= lb$.

Therefore area of border $= (l+2d)(b+2d)-lb$

$= l(b+2d)+2d(b+2d)-lb$

$= 2d(l+b+2d) \ldots (1)$

(*b*) If the border is inside the rectangle, its area $= 2d(l+b-2d) \ldots (2)$

Example 1.—*Find the area of a path* 4 *ft. wide round a rectangular plot; given length* = 25 *ft., breadth* = 14 *ft.*

Turn formula (1) above:
Area = 8(25 + 14 + 8) = 376 sq. ft.

Example 2.—*Find the area of a stained border* 18 *ins. wide round a rectangular room* 18 *ft. long by* 15 *ft. wide.*

From formula (2):
Area = 3(18 + 15 − 3) = 90 sq. ft.

135. The division of one expression by another is indicated in the same manner as in arithmetic, either by the use of the division sign, or by writing the first expression above the second, e.g.:

$3a$ divided by $2b = 3a \div 2b$ or $\dfrac{3a}{2b}$

$(7a + 5b)$ divided by $(3x + 2y)$
$= (7a + 5b) \div (3x + 2y)$
or $\dfrac{7a + 5b}{3x + 2y}$.

136. The division of one power of an expression by another power of the same expression can be performed by subtracting the indices, e.g.:

$$a^5 \div a^3 = \frac{a \times a \times \cancel{a} \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a}} = a^2$$

or $a^5 \div a^3 = a^{5-3} = a^2$.
$8b^4 \div 2b = 4b^{4-1} = 4b^3$,
and in general $x^m \div x^n = x^{m-n}$.

Note.—By the above rule $x^n \div x^n = x^{n-n} = x^0$
But $x^n \div x^n = 1$
$\therefore x^0 = 1$.

This result is of importance in the theory of logarithms.

Examples :

If $a = 6$, $x = 12$, $m = 7$, $p = 4$,
$b = 5$, $y = 10$, $n = 3$, $q = 2$,

then

(a)

$$a^5 \div a^3 = a^2 = 6^2 = 36$$

(b)

$$8b^4 \div 2b = 4b^3 = 4 \times 5 \times 5 \times 5 = 500$$

(c)

$$a^m \div a^n = a^{m-n} = 6^{7-3} = 6 \times 6 \times 6 \times 6 = 1{,}296$$

(d)

$$\frac{(3x + 4y)^p}{(3x + 4y)^q} = (3x + 4y)^{p-q} = (36 + 40)^2 = 76^2 = 5{,}776$$

137. The square root of a number is that number whose square is equal to the given number ;

e.g. square root of 64 = 8, since $8^2 = 64$.

Generally speaking, the nth root of a number is that number whose nth power is equal to the given number. The root of a number is indicated by the symbol $\sqrt{\ }$, the order of the root being shown by a small figure prefixed to the symbol. The square root is indicated, if the symbol stands alone without a prefix, e.g. :

$\sqrt{a^4} = a^2$ since $a^2 \times a^2 = a^4$

$\sqrt[3]{a^{12}} = a^4$,, $a^4 \times a^4 \times a^4 = a^{12}$

$\sqrt[4]{a^{20}} = a^5$,, $a^5 \times a^5 \times a^5 \times a^5 = a^{20}$

From this it is seen that :

$$\sqrt{a^4} = a^{\frac{4}{2}} = a^2$$
$$\sqrt[3]{a^{12}} = a^{\frac{12}{3}} = a^4$$
$$\sqrt[4]{a^{20}} = a^{\frac{20}{4}} = a^5$$

and in general $\sqrt[m]{a^n} = a^{\frac{n}{m}}$

Examples :

(i) $\sqrt{576} = \sqrt{2^6.3^2} = 2^3.3 = 24$

(ii) $\sqrt[3]{512} = \sqrt[3]{2^9} = 2^{\frac{9}{3}} = 2^3 = 8$

(iii) $\sqrt[4]{625} = \sqrt[4]{5^4} = 5.$

(iv) Find the fourth root of 4,100,625.

Factorising, we get $4{,}100{,}625 = 5^4 \times 3^8$

$\therefore \sqrt[4]{5^4 \times 3^8} = 5^{\frac{4}{4}} \times 3^{\frac{8}{4}} = 5 \times 3^2 = 5 \times 9 = 45.$

Examples XXI*a*

(1) If $a = 2$, $b = 3$, $c = 4$, $d = 5$, $e = 6$, find the value of :

(*a*) $2ab + 3bc + 4de$
(*b*) $2a^2 + 3b^2 + 4c^2$
(*c*) $a^3 + b^3 + c^3$
(*d*) $ab(abc + bcd + cde)$
(*e*) $(ab + cd)(ad + bc)$
(*f*) $a^4 \times a^2$
(*g*) $a^2b^3d^2$
(*h*) $a^2b^3 \times a^2d^2$
(*i*) $(4a + 2b + e)^2$
(*j*) $(a^2 + b^2 + c^2 - d^2)^2$
(*k*) $(a^2d^2)^3$
(*l*) $(2a^3 + b^2 - d^2)^5$
(*m*) $(a^2b^2d^2 - c^2d^2)^2$
(*n*) $(ab)^2 - c^2 + d^2e$

(2) If $a = 8$, $b = 3$, $c = 5$, $m = 4$, $n = 2$, find the value of :

(*a*) $(ac)^m \div (ac)^n$
(*b*) $(2a + b^2 + c^2)^n$
(*c*) $abc\,(3a - 7b + c)^{m-n}$
(*d*) $34a + 34b + 34c$
(*e*) $2ab + 2bc$

(3) Find the value of $ma + mb + mc$, if $m = 3{\cdot}6$, $a = 14{\cdot}9$, $b = 28{\cdot}3$, $c = 56{\cdot}8$.

(4) Find the value of:

(*a*) 201^2 (*d*) $(63{\cdot}5)^2 - (36{\cdot}5)^2$ (*g*) $(36{\cdot}9)^2 - (13{\cdot}1)^2$
(*b*) 198^2 (*e*) 122×118 (*h*) $729^2 - 71^2$
(*c*) $1{,}102^2$ (*f*) 202×198 (*i*) $1{,}007 \times 993$.

(5) Find the area of a footpath round a square plot, given:

(*a*) Length of plot = 20 ft., width of path = 2 ft.
(*b*) " " = 30 ft., " " = 30 ins.
(*c*) " " = 45 m., " " = 1·5m.

(6) Given that the area of a border round a rectangular-shaped room can be calculated from the formula, $A = 2d(l + b - 2d)$, find the area when:

(*a*) $l = 22$ ft., $b = 15$ ft., $d = 18$ ins.
(*b*) $l = 18$ ft. 6 ins., $b = 14$ ft. 6 ins., $d = 2$ ft.
(*c*) $l = 6$ metres, $b = 3{\cdot}5$ metres, $d = 5$ decim.

(7) By splitting the following numbers up into prime factors, find their square roots:

(*a*) 1,296 (*c*) 6,561 (*e*) 441 (*g*) 10,816
(*b*) 625 (*d*) 6,084 (*f*) 3,136 (*h*) 7,056

(8) If $a = 3$, $b = 4$, $c = 5$, $m = 2$, $n = 6$, find the value of the following:

(*a*) $\sqrt[m]{81}$, (*c*) $\sqrt[m]{b^4}$, (*e*) $\sqrt[m]{a^4b^2c^6}$.
(*b*) $\sqrt[n]{4096}$, (*d*) $\sqrt[n]{a^{12}}$,

Examples XXI*b*

(1) The time of swing measured in seconds of a pendulum l ft. long, is given by the formula.

$$t = \pi\sqrt{\frac{l}{g}}$$

Given that $\pi = \frac{22}{7}$ and $g = 32$, find the value of t when $l = (a)$ 6 ins., (b) $13\frac{1}{2}$ ins., (c) 24 ins., (d) $37\frac{1}{2}$ ins.

(2) The formula $A = P(1 + \frac{rn}{100})$ represents the amount that £P will amount to at r per cent. in n years, calculating by simple interest.

Find the value of A if—

(a) $P = £250$, $r = 3$, $n = 5$.
(b) $P = £470$, $r = 2\frac{1}{2}$, $n = 4$.
(c) $P = £116\ 4s.$, $r = 3$, $n = 6$.
(d) $P = £25$, $r = 2$, $n = \frac{1}{2}$.

(3) Work the following by contracted methods of multiplication or division correct to 4 places.

(a) Given that $(1{\cdot}03)^{10} = 1{\cdot}3439$, find the value of $(1{\cdot}03)^{20}$.

(b) Given that $(1{\cdot}04)^{5} = 1{\cdot}2167$ and $(1{\cdot}04)^{10} = 1{\cdot}4802$, find the value of $(1{\cdot}04)^{15}$.

(c) Given that $(1{\cdot}05)^{16} = 2{\cdot}1829$ and $(1{\cdot}05)^{12} = 1{\cdot}7959$, find the value of $(1{\cdot}05)^{4}$.

(4) From the formula $(a + b)^2 = a^2 + 2ab + b^2$, find the value of $(7{,}936)^2$, given that $(7{,}935)^2 = 62{,}964{,}225$.

(5) If the canvas surface of a tent can be expressed by the formula $A = \pi r(l + 2h)$, find the number of sq. ft. of canvas, given $\pi = \frac{22}{7}$, $r = 12$ ft., $h = 6$ ft., $l = 14$ ft.

(6) If the time taken by a body to fall s ft. is given by the formula $t = \sqrt{\frac{2s}{g}}$, find the time taken to drop (a) 144 ft., (b) 625 ft., (c) 1,296 ft., given $g = 32$.

(7) The weight of a column of mercury in a capillary tube is given by the formula $W = \pi a^2 h\rho$.

Find the weight in grams if $\pi = \frac{22}{7}$, $a = \cdot 2$, $h = 2\cdot 2$, $\rho = 13\cdot 6$.

(8) The above formula also represents the weight of water in a cylindrical pipe. Find the weight in lbs., given $\pi = \frac{22}{7}$, $a = \frac{1}{4}$, $h = 30$, $\rho = 62\frac{1}{2}$.

(9) Find the value of $(1 + r)^n$ if (a) $r = 3$, $n = 4$, (b) $r = \cdot 03$, $n = 2$.

(10) Find the value of $\dfrac{(1 + \frac{r}{100})^n - 1}{r}$ if $r = 4$, $n = 2$

CHAPTER XXII

MORE ADVANCED AREAS AND VOLUMES, SQUARE ROOT, ETC.

138. IN Chapter IX we dealt with the calculation of areas and volumes as far as they concerned rectangular figures and solids, and in the present chapter it is proposed to extend the discussion so as to include the most common geometrical figures and solids. In a book of this description it would be impossible to give proofs of the formulæ quoted, but they can, if necessary, be verified by reference to books on geometry or mensuration.

139. *Area of a triangle.*

Case (i). If the base and height are known, then area $= \frac{1}{2} \times$ base $\times$ height, so that the area of a triangle $= \frac{1}{2} bh$, where $b =$ number of units of length in the base, $h =$ number of units of length in the height.

Case (ii). If the lengths of the three sides are known,

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where a, b, and c are the lengths of the three sides, and $2s = a + b + c$.

Example 1.—*Find the area of a triangular plot of ground, given the length of one side is* 198 *links, and the distance of the opposite point from this side is* 85 *links.*

Area $= (\frac{1}{2} \times 198 \times 85)$ sq. lks. $=$ 8,415 sq. lks.

Example 2.—*Find the area of a triangular piece of wood, given the lengths of the three sides are* 13, 14, *and* 15 *ins. respectively.*

Using formula (ii), area $= \sqrt{s(s-a)(s-b)(s-c)}$

we find $2s = 13 + 14 + 15 = 42$ ins.

$\therefore s = 21$ ins.

$$\begin{aligned}\text{area} &= \sqrt{21(21-13)(21-14)(21-15)} \text{ sq. ins.}\\ &= \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{3 \times 7 \times 2^3 \times 7 \times 2 \times 3}\\ &= \sqrt{2^4 \times 3^2 \times 7^2}\\ &= 2^2 \times 3 \times 7\\ &= 84 \text{ sq. ins.}\end{aligned}$$

140. The above method of extracting square roots by means of factors cannot be applied to all numbers; so that before proceeding further in the discussion of problems and exercises involving the extraction of square roots, some mention must be made of the general method by which these roots can always be obtained.

141. Since $\sqrt{1} = 1$, $\sqrt{100} = 10$, $\sqrt{10{,}000} = 100$, $\sqrt{1{,}000{,}000} = 1{,}000$:

A number which lies between 1 and 100 has its root between 1 and 10;

A number which lies between 100 and 10,000 has its root between 10 and 100;

A number which lies between 10,000 and 1,000,000 has its root between 100 and 1,000; so that:

A number containing 1 or 2 digits has a root containing 1 digit;

A number containing 3 or 4 digits has a root containing 2 digits;

A number containing 5 or 6 digits has a root containing 3 digits, and so on.

Therefore the number of digits in the root of a given number can be found by ticking off the

number into two-digit periods, commencing from the right and counting an odd digit on the left as a complete period, e.g.:

The square root of 84ˎ56 contains *two* digits.

The square root of 1ˎ53ˎ49 contains *three* digits, and so on.

142. The general rule for extracting the square root of a number can be stated as follows:

(1) Tick the number off into periods in the manner shown in para. 141.

(2) Find the largest square contained in the first period, set it down under this period and subtract. Write down its root as the first figure of the required root.

(3) Bring down the next period; double the part of the root already obtained, and with it make a trial division. Place the quotient obtained by this trial to the right of the divisor, multiply the amended divisor by it, and subtract the product as in division. If the product formed is too large to subtract, choose a smaller figure as quotient until these operations can be performed. Then set down the quotient thus obtained as the next figure of the root.

(4) Repeat the operations given in (3) until all the periods have been brought down. If there is no remainder after the last subtraction, the final result will give the square root required.

Example 1.—*Extract the square root of* 6,889.

```
   |68ˎ89|83
   |64
   |------
163| 489
   | 489
   |------
   | ...
```

(1) The largest square contained in the first period is 64. Subtracting this and bringing down the next period we get remainder 489.

(2) The square root of 64 = 8 (tens).

Required square root = 83

(3) Doubling the 8 (tens) for a divisor we get 16 (tens). This in a trial division goes into 48 three times.

(4) Set down the 3 to the right of the 16 and multiply the 163 thus formed by 3. This gives 489—which leaves no remainder.

(5) Set down the 3 as the second figure of the root.

Example 2.—*Extract the square root of* 126,736.

```
    |12,67,36|356
    | 9
 65 | 367
    | 325
706 |  4236
    |  4236
    |  ....
```

Required square root = 356

In the first division the 6 divides into 36 six times—but the product 6 times 66 is larger than 367, so we must write down the quotient as 5.

143. In finding the square root of a number which is partly integral and partly decimal in form, the periods should be marked in the number, counting from the decimal point outwards in each direction.

Example 1.—*Find the square root of* 1897·4736.

```
     18,97·47,36|43·56
     16
 83  | 297
     | 249
 865 |  48·47
     |  43 25
8706 |   52236
     |   52236
     |   .....
```

$\sqrt{1897·4736} = 43·56.$

Since there are two periods is the integral part of the number, there are two digits in the integral part of the root. The decimal point is therefore placed in the root after the 3.

Example 2.—Extract the square root of 2 (work to three decimal places).

	2·00,00,00	1·4142
	1	
24	100	
	96	
281	400	
	281	
2824	11900	
	11296	
28282	60400	
	56564	
	3836..	

In this and similar instances the decimal periods must be formed by using ciphers.

It is impossible to obtain the exact value of the root, but for most practical purposes the result will be found sufficiently accurate if worked correct to three decimal places.

∴ Correct to the third decimal place $\sqrt{2} = 1·414$.

144. It will be shown in the next chapter that the work of extracting square roots can be greatly simplified by the use of logarithmic tables, so that the above methods are usually employed only when tables are not available.

145. A triangle containing a right angle is known as a right angled triangle, and the side opposite the right angle is called the hypotenuse of the triangle. It can be shown by geometry that the square on the hypotenuse equals the sum of the squares on the other two sides.

Thus, if a = length of the hypotenuse, b and c = length of the other two sides.

$$\text{Then } a^2 = b^2 + c^2.$$

So that if the lengths of the two sides are known, the length of the hypotenuse can be calculated, for,

$$a = \sqrt{b^2 + c^2}$$

Similarly if the lengths of the hypotenuse and one side are known, the length of the other side can be determined, for,

$$b^2 = a^2 - c^2 \quad \therefore b = \sqrt{a^2 - c^2}$$

Example.—A ladder 20 ft. long is placed against the wall of a house so as to reach a window 16 ft. above the ground. How far is the bottom of the ladder from the wall.

The ladder forms the hypotenuse of a right-angled triangle.

$$\begin{aligned} \therefore \text{Required distance} &= \sqrt{20^2 - 16^2} \\ &= \sqrt{400 - 256} \\ &= \sqrt{144} = 12 \text{ ft.} \end{aligned}$$

Examples XXII*a*

(1) A triangular field has its base 158 poles long, and the distance of the opposite corner from this side is 44 poles. What is the rent of the field at £2 10*s*. per acre ?

(2) Find the area of the gable of a house, given that the base is 38 ft. and altitude 22 ft.

(3) Find the square roots of the following :

(*a*) 1,849	(*e*) 99,225	(*h*) 11·9025
(*b*) 2,809	(*f*) 11·56	(*i*) 49·1401
(*c*) 17,424	(*g*) 96·04	(*j*) ·1296
(*d*) 80,656		

(4) Find the area of the following triangles, given the three sides :

(*a*) 28 ft., 18 ft., 34 ft. (*b*) 22 ins., 16 ins., 28 ins. (*c*) 31 yds., 14 yds., 28 yds. (*d*) 24 metres, 63 metres, 44 metres. (*e*) 104 yds., 82 yds., 41 yds.

(5) Right-angled triangles have the lengths of their two sides: (*a*) 10 ft., 25 ft.; (*b*) 130 yds.,

35 yds; (*c*) 144 yds., 108 yds. Find the length of the hypotenuse in each case.

(6) Find the length of the other side of the triangle, if (i) hypotenuse = 25 ft., one side 12 ft.; (ii) hypotenuse = 64 yds., one side 36 yds.; (iii) hypotenuse = 92 m., one side 54 m.

(7) A ladder placed with its foot 5 ft. from the bottom of a wall reaches a point in the wall 12 ft. above the ground. What is the length of the ladder?

(8) Find the area of a triangular field the sides of which are 136 yds., 233 yds., and 283 yds., and its value at £37 10*s*. per acre.

146. A trapezium is a four-sided figure having one pair of opposite sides parallel. Its area is found by the formula:

Area = $\frac{1}{2}$(sum of parallel sides) × (perpendicular distance between them).

Thus, if $ABCD$ is a trapezium having sides AB and DC parallel, then if—

$$\begin{aligned} \text{length of side } AB &= a \text{ units of length,} \\ \text{length of side } DC &= b \text{ units of length,} \\ \text{perpendicular distance } DE &= h \text{ units of length,} \\ \text{area of } ABCD &= \tfrac{1}{2}h(a+b). \end{aligned}$$

Example.—Find the area of a trapezium, given the length of the parallel sides are 18 *ft. and* 24 *ft., and perpendicular distance between them equals* 12 *ft.*

$$\begin{aligned} \text{Area} &= \tfrac{1}{2} \times 12 \times (18 + 24) \text{ sq. ft.} \\ &= 6 \times 42 \text{ sq. ft.} \\ &= 252 \text{ sq. ft.} \end{aligned}$$

147. It can be proved by calculation that the

distance round the edge of a circle is 3·1415926 . . . the greatest distance across it. This is usually stated by formula as :

$$\text{Circumference} = \text{diameter} \times \pi\,;$$

or since the diameter equals twice the radius

$$c = 2\pi r, \text{ when } c = \text{circumference}$$
$$r = \text{radius}$$
$$\text{and symbol } \pi = 3{\cdot}1415926 \ldots$$

The exact value of π cannot be determined, but it can be calculated to any degree of accuracy that may be required. For most purposes, however, it is found sufficiently accurate to use either the value 3·1416, or $3\frac{1}{7}$. The error committed in the latter case is about 4 in 10,000.

Example 1.—*How many complete revolutions are made by a 28-inch cycle wheel in travelling one mile?*

$$\text{Circumference of wheel} = \frac{3{\cdot}1416 \times 28}{36} \text{ yards}$$

$$\text{Distance travelled} = 1{,}760 \text{ yards}$$

$$\therefore \text{No. of revolutions} = \frac{1{,}760 \times 36}{3{\cdot}1416 \times 28} = 720$$

The area of a circle is given by the formula :

$$\text{Area} = \pi r^2.$$

Example 2.—*Forty circular discs of* 3-*inch diameter are stamped from a rectangular sheet of tin,* 24 *ins. long by* 15 *ins. wide. Find the percentage of waste tin.* $\pi = 3\frac{1}{7}$.

Area of :

$$1 \text{ disc} = \tfrac{22}{7} \times 1{\cdot}5 \times 1{\cdot}5 \text{ sq. ins.}$$

$$40 \text{ discs} = \frac{22 \times 3 \times 3 \times 40}{7 \times 2 \times 2} = \frac{1980}{7} = 283 \text{ sq. ins.}$$

Total area of tin $= (24 \times 15) = 360$ sq. ins.
Waste $= (360 - 283) = 77$ sq. ins.

Per cent. of waste $= \dfrac{77 \times 100}{360} = 21{\cdot}4.$

Example 3.—Find to the nearest square foot the area of a footpath 4 ft. wide, round the edge of a circular ornamental lake of 30 yards diameter. $\pi = 3\frac{1}{7}$.

Problems such as the above should be treated in the following manner:

Let r = radius of inner circle.
R = ,, ,, outer ,,

Then area of the ring formed between the two circles

$$= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$$
$$= \pi(R + r)(R - r).$$

Substituting in the above case $r = 45$ ft., $R = 49$ ft.

Area $= 3\frac{1}{7}(49 + 45)(49 - 45) = 3\frac{1}{7} \times 94 \times 4 = 1{,}182$ sq. ft.

EXAMPLES XXII*b* ($\pi = 3\frac{1}{7}$)

(1) Find the area of circles with radii (*a*) 6 ins., (*b*) 8 ins., (*c*) 3 ft.

(2) What is the radius of a circle which has an area of 30 sq. ins.?

(3) A cyclist notices that his front wheel revolves 720 times between two consecutive mile posts. What is the diameter of the wheel?

(4) Find the speed in miles per hour with which a driving belt travels, if it passes round a wheel of 20 ins. radius making 120 revolutions per minute (correct to first decimal place).

(5) Find the area of a circular plot of ground of 10 ft. radius. What is the area of a footpath 3 ft. wide built round this plot?

(6) Find the cost of laying a circular footpath 4 ft. wide round a fountain of 30 ft. diameter. Given the cost per sq. ft. = 2*s*. 3*d*.

(7) What is the area of a circle which has its circumference 30 ft.? Find the area of a square having the same length for its perimeter.

(8) A square enclosure is formed by 400 ft. of fencing. Find the number of square feet gained by using the fencing to form a circular enclosure.

148. It was stated in para. 72 that the volume of a cuboid can be calculated from the formula :

$$\text{Volume} = \text{area of base} \times \text{height}.$$

Similarly the area of the walls can be found by using the formula :

$$\text{Area of walls} = \text{perimeter} \times \text{height}.$$

Both these formulæ are applicable to all right prisms, including cylinders, so that if the area of the base of a prism can be calculated, the cubical contents can also be found. Similarly, knowing the perimeter, we are able to calculate the area of the walls.

In the particular case of a cylinder, if r = radius of base, and h = height of the cylinder,

$$\text{then area of base} = \pi r^2$$
$$\text{and volume of cylinder} = \pi r^2 h.$$

Again, the perimeter in this case is the circumference of the base, so that perimeter—

$$= 2\pi r.$$

and area of curved walls

$$= 2\,\pi\,r\,h.$$

The total area of the walls and ends therefore—

$$= 2\pi rh + 2\pi r^2$$
$$= 2\pi r(h + r).$$

149. The cubical contents of a hollow cylinder can also be calculated by using the above formula, in which case the area of the base is calculated in the manner shown in Example 3, so that the volume of a hollow cylinder

$$= \pi(R^2 - r^2)\,h \text{ or } \pi(R + r)\,(R - r)h.$$

Example 1.—*Find the cost of excavating a V-shaped trench* 120 *yards long at* 1s. 6d. *per cubic yard, given that the width of the top of the trench is* 3 *ft.* 6 *ins. and depth is* 28 *ins.*

Cross-section of the trench is a triangle.

Area of this triangle $= \frac{1}{2} \times 3\frac{1}{2} \times 2\frac{1}{3}$ sq. ft.

$$= \frac{7 \times 7}{2 \times 2 \times 3 \times 9} \text{ sq. yds.}$$

$$= \tfrac{49}{108} \text{ sq. yds.}$$

Volume excavated $= (\frac{49}{108} \times 120) =$ cubic yds.

Cost of excavation $= \frac{49}{108} \times 120 \times \frac{3}{2}$ shillings

$= £4\ 1s.\ 8d.$

Example 2.—*How many iron pipes, each* 9 *ft. long, can be loaded into a truck made to carry* 14 *tons, given that the inner diameter of a pipe is* 8 *ins., outer diameter* $9\frac{1}{2}$ *ins., and weight of iron* 480 *lbs. per cubic foot* ($\pi = 3\frac{1}{7}$)?

Volume of a hollow cylinder $= \pi(R + r)\,(R - r)h$.

Outer radius $= 4\frac{3}{4}$ ins. Inner radius $= 4$ ins.

$$\therefore \text{Volume of each pipe} = \frac{22}{7} \times \frac{8\frac{3}{4}}{12} \times \frac{\frac{3}{4}}{12} \times 9 \text{ cubic ft.}$$

$$\therefore \text{Weight of one pipe} = \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\overset{5}{\cancel{35}}}{\cancel{48}} \times \frac{1}{\cancel{16}} \times 9 \times \underset{4}{\overset{105}{\cancel{480}}} \text{ lbs.}$$

$$= \tfrac{2475}{4} \text{ lbs.}$$

$$\therefore \text{No of pipes to weigh 14 tons} = \frac{14 \times 2{,}240 \times 4}{2{,}475}$$

$\therefore$ No. of pipes in a load $= 50$.

EXAMPLES XXII*c*

(1) Find the volume of a right prism of height 10 ins., given the base is a triangle with sides 6, 7, and 8 ins. respectively.

(2) Find the number of cubic feet of timber in a cylindrical log of length 10 ft., diameter 8 ft.

(3) A garden roller has a diameter of 2 ft. 6 ins., and width 3 ft. Find the number of square feet rolled over in 12 complete revolutions.

(4) Six borings are made in a certain iron casting. Find the weight of the material removed if four of the borings are 6 ins. deep and have 2 ins. radii, while two are 8 ins. deep and have 3 ins. radii. Weight of casting = 480 lbs. per cubic foot.

(5) Find the number of gallons contained in a cylindrical tank of 3 ft. diameter and 6 ft. high.

(6) A trench is dug so that one side is 4 ft. deep and the other 6 ft. deep, the distance between them being 3 ft. Find the number of cubic yards of soil excavated per 100 yards length.

(7) Find the cubical contents of a haystack which has its ends in the form of a rectangle surmounted by a triangle. Given the height of the eaves from the ground is 10 ft., the height of the ridge 15 ft., width of stack 12 ft., and length 20 ft.

(8) A person wishing to find the average thickness of a lead pipe, notices that 1 gallon of water fills a length of 21 ft. If this length of piping weighs 138 lbs. and the sp. gr. of lead = 11·3, find the thickness of the lead.

(9) An open cylindrical iron vessel of 8 cms. external radius and 30 cms. height is placed in water and found to sink to a depth of 12 cms.

Find the thickness of the walls, given the sp. gr. of iron = 7·8.

150. The volume of a sphere is given by the formula :

$$V = \tfrac{4}{3}\pi r^3, \text{ where } r = \text{radius of sphere.}$$

The surface of a sphere is given by :

$$S = 4\pi r^2.$$

Example.—Find the surface and volume of a sphere of 8 *cms. radius.* $\pi = 3{\cdot}1416$. *Work correct to nearest unit.*

$$S = 4\pi r^2 = 4 \times 3{\cdot}1416 \times 8^2$$
$$= 3{\cdot}1416 \times 256 = 80 \text{ sq. cm.}$$
$$V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3} \times 3{\cdot}1416 \times 8^3$$
$$= \frac{3{\cdot}1416 \times 2048}{3} = 2145 \text{ c.c.s.}$$

EXAMPLES XXII*d*

Unless otherwise stated, take $\pi = 3\tfrac{1}{7}$.

(1) Find to the nearest square and cubic cm., the surfaces and volumes of spheres having radii : (*a*) 10 cm. (*b*) 21 cm. (*c*) 1·25 metres. ($\pi = 3{\cdot}1416$.)

(2) An 11-inch cube of stone is carved into an ornamental stone sphere of 10-ins. diameter. Find the percentage of waste.

(3) A hemispherical dome of 10 ft. radius is covered with sheet lead at a cost of 2*s.* 6*d.* per square foot. Find the total cost.

(4) How many lead shot of $\tfrac{1}{4}$ in. diameter can be cast from 10 lbs. of lead (1 cubic foot of lead weighs 710 lbs.).

(5) Find the weight of a hollow cast-iron ball, given the outer diameter is 10 ins., thickness of iron is $\frac{1}{2}$ in., and weight of cubic foot of iron is 450 lbs.

(6) What is the capacity (in pints) of a hemispherical bowl which has an internal radius of 8 ins. (see para. 74).

(7) A hemispherical bowl of lead which has its inner radius 8 ins. and thickness of lead $\frac{1}{5}$ in., is set to float in water. Is this possible? If so, how many pints of water must be poured into it before it will sink? Sp. gr., lead = 11·35 (see para. 74).

151. The volume of any pyramid or cone = $\frac{1}{3}$ × area of base × height of vertex above the base (= $\frac{1}{3}$ × volume of cylinder of same height and on same base).

In a right circular cone

$$\begin{aligned} \text{if } r &= \text{radius of base,} \\ h &= \text{height of vertex,} \\ l &= \text{length of sloping side,} \\ \text{then volume} &= \tfrac{1}{3}\pi r^2 h. \end{aligned}$$

The area of sloping surface

$$\begin{aligned} &= \tfrac{1}{2} \text{ circumference of base} \times \text{slant height} \\ &= \tfrac{1}{2} \times 2\pi r l = \pi r l \\ \text{or } &= \pi r\sqrt{h^2 + r^2}. \end{aligned}$$

The total surface of the cone

$$\begin{aligned} &= \text{curved surface} + \text{area of base} \\ &= \pi r l + \pi r^2 \\ &= \pi r(l + r). \end{aligned}$$

Example.—A semicircular piece of tin of 10 *ins. radius is rolled into a cone having the centre of the circle as its vertex. Find the area of the circular*

piece of tin needed to form a base. What is the height of the cone?

The edge of the semicircle will form the circumference of the base.

$$\therefore\ 10\pi = 2\pi r \text{ where } r = \text{radius of base,}$$
$$\text{or } r = 5 \text{ ins.}$$
$$\text{Area of base} = \pi r^2 = 3\tfrac{1}{7} \times 25 \text{ sq. ins.}$$
$$\text{(i)} = 78{\cdot}5 \ldots \text{ sq. ins.}$$
$$\text{The slant edge of cone} = 10 \text{ ins.}$$
$$\text{radius of base} = 5 \text{ ins.}$$
$$\text{but } l^2 = h^2 + r^2\ \therefore\ h^2 = l^2 - r^2$$
$$\text{or } h^2 = 10^2 - 5^2$$
$$\therefore\quad h = \sqrt{75}$$
$$= 8{\cdot}7 \text{ ins. approx.}$$

EXAMPLES XXIIe

(1) Find the area of the canvas required for a conical tent which has slant height 12 ft., radius of base 6 ft. What is the height of the central pole?

(2) What is the capacity in pints of a funnel (neglecting the spout), given that depth is 5 ins. and diameter of top 6 ins.?

(3) A funnel capable of holding 1 quart has a radius of 3 ins. at its brim. Find its depth correct to the nearest tenth of an inch.

(4) A circular tent of 20 ft. diameter and 18 ft. high has vertical walls 3 ft. in height. Find the amount of canvas used in its construction, allowing 10 per cent. waste.

CHAPTER XXIII

LOGARITHMS

152. LET N and M represent two numbers which can be expressed as powers of a certain base, a.

So that, $M = a^x$, $N = a^y$.

Then, as was shown in paras. 130, 131, 136, 137 :

$$M \times N = a^x \times a^y = a^{x+y}$$
$$M \div N = a^x \div a^y = a^{x-y}$$
$$(M)^q = (a^x)^q = a^{qx}$$
$$\sqrt[p]{M} = \sqrt[p]{a^x} = a^{\frac{x}{p}}$$

From these results it is seen that the operation of multiplication when performed by means of indices is simplified into one of addition, similarly division is simplified into subtraction, and so on. If, therefore, we express all numbers as powers of the same base, the above methods can be generally applied, thus resulting in a considerable simplification in the processes of numerical calculation; e.g.

Consider the method by which the following simple table has been constructed and the manner in which certain calculations can be performed by means of it.

$1 = 2^0$	$32 = 2^5$	$1{,}024 = 2^{10}$	$32{,}768 = 2^{15}$
$2 = 2^1$	$64 = 2^6$	$2{,}048 = 2^{11}$	$65{,}536 = 2^{16}$
$4 = 2^2$	$128 = 2^7$	$4{,}096 = 2^{12}$	$131{,}072 = 2^{17}$
$8 = 2^3$	$256 = 2^8$	$8{,}192 = 2^{13}$	$262{,}144 = 2^{18}$
$16 = 2^4$	$512 = 2^9$	$16{,}384 = 2^{14}$	$524{,}288 = 2^{19}$

Example 1.—*Multiply* 2,048 *by* 128.

2,048	$= 2^{11}$
128	$= 2^{7}$
$\therefore$ product $2^{11} \times 2^{7}$	$= 2^{18}$
From the tables 2^{18}	$= 262,144$
$\therefore$ product	$= 262,144$

Example 2.—*Divide* 131,072 *by* 4,096.

131,072	$= 2^{17}$
4,096	$= 2^{12}$
$\therefore$ quotient	$= 2^{17} \div 2^{12}$
	$= 2^{17-12}$
	$= 2^{5}$
From the tables 2^{5}	$= 32$
$\therefore$ quotient	$= 32.$

Example 3.—*Find the square of* 256.

256	$= 2^{8}$
$\therefore (256)^2$	$= 2^{8 \times 2}$
	$= 2^{16}$
From the tables 2^{16}	$= 65,536$
$\therefore$ square of 256	$= 65,536.$

Example 4.—*Find the square root of* 262,144.

262,144	$= 2^{18}$
$\therefore \sqrt{262,144}$	$= 2^{\frac{18}{2}} = 2^{9}$
From the tables 2^{9}	$= 512$
$\therefore$ square root	$= 512.$

EXAMPLES XXIII*a*

By using the table in a manner similar to the above, obtain the answers to the following:

(1) $8,192 \times 64$

(2) $32,768 \times 16$

(3) $512 \times 64 \times 16$

(4) $131,072 \div 8,192$

(5) $32{,}768 \div 1{,}024$	(9) 16^4
(6) $524{,}288 \div 2{,}048$	(10) $\sqrt{65{,}536}$
(7) $(512)^2$	(11) $\sqrt[3]{262{,}144}$
(8) $(64)^3$	(12) $\sqrt[5]{32{,}768}$

153. In the above examples the working is carried out by means of the indices only, so that as far as the actual calculations go, the base itself can be omitted.

This relative importance of indices and base is indicated better if the tables are constructed so that the indices are written apart from their base, in which form they are referred to as logarithms.

Thus instead of writing $256 = 2^8$, we can write its equivalent form: $\log_2 256 = 8$.

This is read: the logarithm of 256 to the base 2 equals 8.

It should be emphasised at this stage that when in future we refer to a logarithm, we imply nothing more than an index, so that the laws governing the use of logarithms are exactly the same as those given for indices in paras. 130, 131, 136, 137.

Written in logarithmic form, the table given in para. 152 is as follows:

$\log_2 1 = 0$	$\log_2 1{,}024 = 10$
$\log_2 2 = 1$	$\log_2 2{,}048 = 11$
$\log_2 4 = 2$	$\log_2 4{,}096 = 12$
$\log_2 8 = 3$	$\log_2 8{,}192 = 13$
$\log_2 16 = 4$	$\log_2 16{,}384 = 14$
$\log_2 32 = 5$	$\log_2 32{,}768 = 15$
$\log_2 64 = 6$	$\log_2 65{,}536 = 16$
$\log_2 128 = 7$	$\log_2 131{,}072 = 17$
$\log_2 256 = 8$	$\log_2 262{,}144 = 18$
$\log_2 512 = 9$	$\log_2 524{,}288 = 19$

154. Before working out examples by means of

logarithmic tables, the index laws may be restated in their logarithmic form, thus:

If all numbers can be expressed as powers of a certain base, then:

Theorem 1.—Since the index of a product is formed by adding the indices of the factors,

The logarithm of a product is formed by adding the logarithms of the factors.

e.g. $256 = 2^8$ or $\log_2 256 = 8$
$128 = 2^7$ or $\log_2 128 = 7$

$\therefore (256 \times 218) = 2^{8+7}$ or $\log_2(256 \times 128) = 7 + 8$

i.e $\log (256 \times 128) = \log 256 + \log 128$,

and in general $\log (mn) = \log m + \log n$;

similarly $\log (mnp) = \log m + \log n + \log p$.

Theorem 2.—Since the index of a quotient is formed by subtracting the index of the divisor from the index of the dividend, therefore:

The logarithm of a quotient is formed by subtracting the logarithm of the divisor from the logarithm of the dividend;

e.g. $2{,}048 = 2^{11}$ or $\log_2 2{,}048 = 11$

$256 = 2^8$ or $\log_2 256 = 8$,

$\therefore (2{,}048 \div 256) = 2^{11-8}$ or $\log_2 (2{,}048 \div 256) = 11 - 8$,

i.e. $\log (2{,}048 \div 256) = \log 2{,}048 - \log 256$,

and in general $\log (\frac{m}{n}) = \log m - \log n$.

Theorem 3.—Since the index of a power is formed by multiplying the index of the original number:

The logarithm of a power is formed by multiplying the logarithm of the original number;

e.g. $512 = 2^9$ or $\log_2 512 = 9$

and $(512)^2 = 2^{9\times2}$ or $\log_2 (512)^2 = 9 \times 2$

$\therefore \log_2 (512)^2 = 2 \times \log_2 512$,

and in general $\log m^n = n \log m$.

Theorem 4.—Since the index of a root is formed by dividing the index of the original number, so

The logarithm of a root is formed by dividing the logarithm of the original number;

e.g. $32{,}768 = 2^{15}$ or $\log_2 32{,}768 = 15$

and $\sqrt[3]{32{,}768} = 2^{\frac{15}{3}}$ or $\log_2 \sqrt[3]{32{,}768} = \frac{15}{3}$

i.e. $\log_2 \sqrt[3]{32{,}768} = \frac{1}{3} \log_3 32{,}768$,

and in general $\log \sqrt[n]{m} = \frac{1}{n} \log m$.

155. The number which is represented by a given logarithm is called its *antilogarithm*;

e.g. since $8 = \log_2 256$

then $256 =$ antilogarithm of 8, or as is more usually written $=$ antilog 8.

156. The four examples worked out in para. 152 appear as follows when worked by means of the table in para. 153.

(1) *Multiply* 2,048 *by* 128.

$$\begin{aligned} \log_2 2{,}048 &= 11 \\ \log_2 \quad 128 &= 7 \\ \hline \log_2 \text{product} &= 18 \text{ (Rule 1)} \\ \text{antilog}_2\, 18 &= 262{,}144 \end{aligned}$$

(2) *Divide* 131,072 *by* 4,096.

$$\begin{aligned} \log_2 131{,}072 &= 17 \\ \log_2 4{,}096 &= 12 \\ \hline \log_2 (\text{quotient}) &= 5 \text{ (Rule 2)} \\ \text{antilog}_2\, 5 &= 32 \end{aligned}$$

(3) *Find the square of* 256.

$$\begin{aligned} \log_2 256 &= 8 \\ \therefore \log_2 (256)^2 &= 16 \text{ (Rule 3)} \\ \text{antilog } 16 &= 65{,}536 \end{aligned}$$

(4) *Find the square root of* 262,144.

$$\log_2 266{,}144 = 18$$
$$\therefore \log_2 \sqrt{262{,}144} = 9 \text{ (Rule 4)}$$
$$\text{antilog } 9 = 512$$

The following example illustrates all the rules together:

(5) *Find the value of* $\sqrt[4]{\dfrac{64^2 \times 2{,}048 \times 1{,}024}{131{,}072}}$

$$\begin{aligned}
\log_2 64 &= 6 \\
\therefore \log_2 64^2 &= 12 \text{ (Rule 3)} \\
\log_2 2{,}048 &= 11 \\
\log_2 1{,}024 &= 10 \\
\log \text{(product)} &= 33 \text{ (Rule 1)} \\
\log 131{,}072 &= 17 \\
\log \text{(quotient)} &= 16 \text{ (Rule 2)}
\end{aligned}$$

The fourth root of quotient $= \frac{16}{4} =$ 4 (Rule 4)

antilog 4 = 16

$\therefore$ required value = 16.

EXAMPLES XXIII*b*

(1) Work out the exercises in Example XXIII*a* by the means of the log. table of para. 153, in the manner shown above.

(2) Using the same table, find the value of each of the following:

(*a*) $\dfrac{131{,}072 \times 8{,}192 \times 512}{524{,}288 \times 32{,}768}$

(*b*) $\left(\dfrac{131{,}072}{2{,}048}\right)^3$

(*c*) $\sqrt[5]{\dfrac{262{,}144 \times 16{,}384}{131{,}072}}$

(*d*) $\sqrt[3]{\dfrac{262{,}144 \times 16{,}384 \times 4{,}096}{131{,}072}}$

157. Any base can be chosen upon which to build a system of logarithms, but in practice it has been found most convenient to use the number 10 for this purpose. Up to the present we have only considered logarithms which were integers, but it is obvious that only a very limited number of logarithms can be integers when calculated to the base 10. For example, in the range of numbers 1 to 10,000 there are only four integral logarithms—as follows:

$$\begin{aligned} 10{,}000 &= 10^4 \therefore \log 10{,}000 = 4 \\ 1{,}000 &= 10^3 \therefore \log 1{,}000 = 3 \\ 100 &= 10^2 \therefore \log 100 = 2 \\ 10 &= 10^1 \therefore \log 10 = 1 \\ 1 &= 10^0 \therefore \log 1 = 0 \end{aligned}$$

Since $\log 1 = 0$, and $\log 10 = 1$, the logarithms of numbers between 1 and 10 must be purely fractional or decimal in form, similarly the logs of numbers between 10 and 100 must be 1 plus a fraction, between 100 and 1,000, 2 plus a fraction, and so on. The fractional or decimal part of a logarithm is called the *mantissa*, and the integral part the *characteristic*.

158. As will be shown in para. 160, only the mantissa of a logarithm need be calculated, as the characteristic can always be found by inspection. The actual process of calculating the mantissa is beyond the scope of this book, but the results can be obtained for practical use in four-, five-, or seven-figure tables. For general purposes the four-figure tables will be found sufficiently correct, but for more accurate work seven-figure tables should be consulted. Before proceeding to the description of and method of using four-figure

tables, it will probably be found advantageous to obtain some idea of what is meant by a decimal form of logarithm. This can readily be obtained by studying the following examples:

Thus $10^{\cdot 5} = 3{\cdot}162\ldots$ since $10^{\cdot 5} = 10^{\frac{1}{2}} = \sqrt{10}$

$10^{\cdot 25} = 1{\cdot}778\ldots$
$10^{\cdot 125} = 1{\cdot}334\ldots$
$10^{\cdot 0625} = 1{\cdot}154\ldots$

Each index is half the preceding index—so that each number is the square root of the preceding number.

Again $10^{1\cdot 5} = 31{\cdot}62\ldots$ since $10^{1\cdot 5} = 10^{\frac{3}{2}} = \sqrt{1000}$

$10^{\cdot 75} = 5{\cdot}623\ldots$
$10^{\cdot 375} = 2{\cdot}371\ldots$
$10^{\cdot 1875} = 1{\cdot}540\ldots$

Derived in the same manner as the above.

159. These results expressed in logarithmic form to the base 10 are therefore as follows:

$\log 3{\cdot}162\ldots = {\cdot}5$	$\log 31{\cdot}62\ldots = 1{\cdot}5$
$\log 1{\cdot}778\ldots = {\cdot}25$	$\log 5{\cdot}623\ldots = {\cdot}75$
$\log 1{\cdot}334\ldots = {\cdot}125$	$\log 2{\cdot}371\ldots = {\cdot}375$
$\log 1{\cdot}154\ldots = {\cdot}0625$	$\log 1{\cdot}540\ldots = {\cdot}1875$

It will be observed that the base has been omitted when writing the above logarithms. This is the general procedure when the base is 10, though other bases must always be indicated.

From the above table, find the value of:

(*a*) $3{\cdot}162 \times 1{\cdot}778$

$\log 3{\cdot}162 = {\cdot}5$
$\log 1{\cdot}778 = {\cdot}25$

antilog ${\cdot}75 = 5{\cdot}623$

(*b*) $\sqrt[3]{2{\cdot}371}$

$\log 2{\cdot}371 = {\cdot}375$
$\therefore \log \sqrt[3]{2{\cdot}371} = {\cdot}125$ dividing by 3
antilog ${\cdot}125 = 1{\cdot}334$

EXAMPLES XXIIIc

In the same manner as the above, find the value of:

(*a*) $1{\cdot}778 \times 1{\cdot}334$

(*b*) $1{\cdot}334 \times 1{\cdot}154$

(*c*) $(3{\cdot}162)^3$

(*d*) $2{\cdot}371 \div 1{\cdot}334$

(*e*) $5{\cdot}623 \div 1{\cdot}778$

(*f*) $\dfrac{2{\cdot}371 \times 3{\cdot}162}{5{\cdot}623}$

(*g*) $(1{\cdot}154)^2 \times 2{\cdot}371$

(*h*) $\sqrt[3]{5{\cdot}623}$

(*i*) $\sqrt[4]{3{\cdot}162 \times 1{\cdot}778}$

160. The actual relation between the characteristic and mantissa of a logarithm can easily be determined by considering the following results:

$\log (1{,}778) = \log (1{\cdot}778 \times 1{,}000) = \log 1{\cdot}778 + \log 1{,}000 = {\cdot}25 + 3 = 3{\cdot}25.$

$\log (177{\cdot}8) = \log (1{\cdot}778 \times 100) = \log 1{\cdot}778 + \log 100 = {\cdot}25 + 2 = 2{\cdot}25.$

$\log (17{\cdot}78) = \log (1{\cdot}778 \times 10) = \log 1{\cdot}778 + \log 10 = {\cdot}25 + 1 = 1{\cdot}25$

$\log (1{\cdot}778) = {\cdot}25.$

$\log ({\cdot}1778) = \log (1{\cdot}778 \div 10) = \log 1{\cdot}778 - \log 10 = {\cdot}25 - 1 = \bar{1}{\cdot}25.$

$\log ({\cdot}01778) = \log (1{\cdot}778 \div 100) = \log 1{\cdot}778 - \log 100 = {\cdot}25 - 2 = \bar{2}{\cdot}25.$

$\log ({\cdot}001778) = \log (1{\cdot}778 \div 1{,}000) = \log 1{\cdot}778 - \log 1{,}000 = {\cdot}25 - 3 = \bar{3}{\cdot}25.$

From this it is seen that the mantissa of the logarithm of a number depends only on *the order of the digits* in the number, while the characteristic is the index of a power of ten. The rules for finding the characteristic by inspection are as follows:

Rule 1.—*If the number is greater than unity, the characteristic is one less than the number of*

digits to the left of the decimal point, and is positive.

Rule 2.—*If the number is less than unity, the characteristic is one more than the number of ciphers lying immediately to the right of the decimal point and is negative.*

These rules will be seen to be simply deductions made from the results stated above.

161. When writing down the logarithm of a number the characteristic should always be written first, as there is a tendency on the part of beginners to overlook it. The mantissa is then obtained from tables and written after the decimal point, so as always to be positive. If it gives a negative result, as in the case of proper fractions, the following method is adopted for changing it into a positive mantissa with a negative characteristic. In order to keep the mantissa positive the negative sign of a characteristic is written above it instead of in front of it.

Thus $\bar{3}\cdot25$ indicates $-3 + \cdot25$.

While $-3\cdot25$ would indicate $-3 - \cdot25$.

Example 1.—*Write down the logarithm of* $\frac{1}{5}$.

$$\begin{aligned}\log \tfrac{1}{5} &= \log 1 - \log 5\\ &= 0 - \cdot6990 \text{ (from tables)}\\ &= -1 + 1 - \cdot6990\\ &= \bar{1}\cdot3010\end{aligned}$$

162. The following examples should be carefully studied, as they illustrate the manner in which logarithms are written and used.

Example 2.—*Find the log of* $(\cdot01334 \times 56\cdot23)$.

$$\begin{aligned}\log \cdot01334 &= \bar{2}\cdot125\\ \log 56\cdot23 &= 1\cdot75\\ \hline \log \text{(product)}\ &\bar{1}\cdot875 \text{ by addition.}\end{aligned}$$

Example 3.—*Find the log of* (1·778 ÷ 562·3).

$$\begin{aligned} \log 1{\cdot}778 &= \ {\cdot}25 \\ \log 562{\cdot}3 &= 2{\cdot}75 \\ \log (\text{quotient}) &= \bar{3}{\cdot}50 \text{ by subtraction.} \end{aligned}$$

Example 4.—*Find the log of* (1·334 ÷ ·002371).

$$\begin{aligned} \log 1{\cdot}334 &= \ {\cdot}125 \\ \log {\cdot}002371 &= \bar{3}{\cdot}375 \\ \log (\text{quotient}) &= 2{\cdot}875 \text{ by subtraction.} \end{aligned}$$

Subtracting a negative characteristic is equivalent to adding a positive characteristic to the top line.

Example 5.—*Find the log of* $\sqrt[5]{{\cdot}05623}$.

$$\begin{aligned} \log \ {\cdot}05623 &= 2{\cdot}75 \\ \therefore \log \sqrt[5]{{\cdot}05623} &= \tfrac{1}{5}(\bar{2}{\cdot}75) \\ &= \tfrac{1}{5}(5 + 3{\cdot}75) \\ &= 1{\cdot}75 \end{aligned}$$

EXAMPLES XXIII*d*

(1) Write down the characteristics of the logarithms of:
(*a*) 317·2. (*b*) ·3172. (*c*) 31·72. (*d*) 317200. (*e*) ·003172.

(2) Given log 5·632 = 0·757, write down the logarithms of:
(*a*) 56·32. (*b*) ·05632. (*c*) 5,632. (*d*) ·0005632.

(3) Given log 4·215 = ·6248, write down the antilogarithms of:
(*a*) 1·6248. (*b*) 2·6248. (*c*) 2·6248. (*d*) $\bar{3}$·6248.

(4) Rewrite the following logarithms so as to have a positive mantissa:

(*a*) − ·4152. (*c*) − 5·3215.
(*b*) − 2·3215. (*d*) − 6·6248.

(5) Simplify the following:

(a) $\bar{7}\cdot6423 + 3\cdot4156$. (e) $3\cdot6415 - 2\cdot7483$.
(b) $3\cdot2145 + \bar{5}\cdot3281$. (f) $\bar{6}\cdot4234 - \bar{4}\cdot3821$
(c) $\bar{5}\cdot7821 + 1\cdot3492$. (g) $\bar{2}\cdot4915 - \cdot6421$.
(d) $4\cdot6281 + \bar{3}\cdot7294$. (h) $\cdot6243 - 3\cdot4812$.

(6) Find the value of:

(a) $\bar{4}\cdot6381 \times 3$. (d) $2\cdot5842 \times 4$.
(b) $\bar{2}\cdot3842 \times 6$. (e) $\bar{1}\cdot9432 \times 8$.
(c) $1\cdot6324 \times 2$.

(7) Find (correct to the fourth decimal place) the value of:

(a) $\bar{3}\cdot4287 \div 5$. (d) $2\cdot3148 \div 6$.
(b) $\bar{4}\cdot6239 \div 4$. (e) $3\cdot2148 \div 8$.
(c) $\bar{2}\cdot4835 \div 3$.

(8) Given $\log 1\cdot05 = \cdot0211893$, find the value of (i) $\log (1\cdot05)^{22}$; (ii) $\log \dfrac{1}{(1\cdot05)^{30}}$.

(9) Given $\log 106 = 2\cdot0253059$, find the value of (i) $\log (106)^{3}$; (ii) $\log \dfrac{1}{(10\cdot6)^{4}}$.

THE USE OF LOGARITHM TABLES

163. Since the characteristic of a logarithm can always be found by inspection, only the mantissa need be tabulated. The method of reading the tables and finding the mantissa will be easily understood by considering the following example:

Example 1.—*Find the value of log* 215.

The first two digits are 21, therefore looking down the first column for this row, we find the following:

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18

Since the third digit is 5, we look along the row for column 5, and find the mantissa of 215 is 3324, $\therefore$ log 215 = 2·3324.

Example 2.—*Find the value of log* 2,156.

Proceed as above to obtain the mantissa corresponding to the first three figures; and then, from the table of differences at the end, take the number from the columns headed by the fourth figure, 6. Add this number 12 to the mantissa already found.

We get mantissa 2150 = 3324
difference for 6 = 12

$\therefore$ mantissa for 2156 = 3336

$\therefore$ log 2156 = 3·3336.

When actually using tables, the difference will, of course, be added mentally.

164. Having found the logarithms and used them to perform the various calculations, it is necessary to find the antilogarithm of the resulting logarithm. This can be done either by means of the ordinary logarithm tables or by using antilogarithm tables.

In the first case we look in the body of the tables for the mantissa nearest to the one in hand; note the difference between the two, and then look for this number in the table of differences. The first reading gives us the first three digits of the antilogarithm; and the last reading the correction which must be applied.

The position of the decimal point in the antilogarithm obtained is fixed as follows:

The characteristic increased by 1 *if positive gives the number of figures to the left of the decimal point,*

and diminished by 1 *if negative gives the number of ciphers to the right of the decimal point.*

Example.—Find antilog $\bar{3}$·3368, *using ordinary log tables.*

In the extract shown in para. 163 the nearest mantissa to ·3368 is ·3365.

An addition of 3 must be made—and this from the table of differences indicates an addition of either 1 or 2 to the antilog.

But antilog ·3365 = 2·17

∴ antilog ·3368 = 2·171 or 2·172

∴ antilog $\bar{3}$·3368 = ·002171 or ·002172.

165. *Find antilog* 2·4321 *by means of antilogarithm tables.*

Looking down the tables for the row containing ·43 in the first column, we select the number in the column headed 2. This gives 2704. The correction to be added to this on account of the fourth figure, 1, is found from the table of differences to be 1,

∴ order of digits in antilog = 2704 + 1 = 2705.

∴ antilog 2·4321 = 270·5.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·3	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6

Practice in the use of both logarithms and antilogarithm tables can be obtained by taking any four digit numbers and finding their logarithms. Then reference to the antilogarithm tables with the logarithm found should give the original number again. This procedure will give an unlimited amount of practice in which the student can check his own work.

CHAPTER XXIV

COMPOUND INTEREST

166. In calculating simple interest on a given sum of money it is assumed that the borrower pays to the lender each instalment of interest as it becomes due. If this is not so, then his liability is increasing, for he is borrowing to a still greater extent by keeping the interest, and he should be expected to pay interest on the extra sum borrowed.

When money is lent out in such a manner that no interest is paid as it becomes due, but is added to the principal, the latter is said to be accumulating at compound interest. In this case the principal is continually being increased, and the interest for each period must be calculated on the amount at the end of the preceding period. The manner in which both principal and interest increase are best studied by means of actual examples.

Unless otherwise stated, it is assumed that the interest is payable yearly.

Example 1.—*Find to the nearest penny the compound interest on* £450 *for* 3 *years at* 3 *per cent.*

1st principal	£450·
1st interest	13·50
2nd principal	463·50
2nd interest	13·905
3rd principal	477·405
3rd interest	14·32215
Amount in 3 years	491·72715
Original principal	450·
Interest required	41·727
£41·727 =	£41 14*s.* 7*d.*

In calculating the interest at 3 per cent., multiply the principal by 3, commencing to write the result, however, two places to the right.

Example 2.—*Find to the nearest penny the compound interest on* £834 13*s.* 5*d. for* 3 *years at* 4 *per cent.*

1st principal	£834·670|83
1st interest	33·386|83
2nd principal	868·057|66
2nd interest	34·722|30
3rd principal	902·779|96
3rd interest	36·111|20
Amount in 3 years	938·891|..

£938·891 =	£938	17	10
Original principal =	834	13	5
	£104	4	5

Work to five decimal places to get the result correct to the third. The interest is obtained as in Example 1 by multiplying the principal by 4 and commencing the result two places to the right. Since no figure is required after the fifth decimal place, it will be sufficient to commence multiplying each time at the third decimal place

figure. If a vertical line is drawn as shown, the point at which to commence multiplying can readily be obtained each time.

Example 3.—Find to the nearest penny the amount of £341 5s. 5d. for $2\frac{1}{2}$ years at $4\frac{3}{4}$ per cent. compound interest.

	£341·270	83	
Interest at 4%	13·650	83	interest for 1st year.
$\frac{1}{2}\% = \frac{1}{8}$ of 4%	1·706	35	
$\frac{1}{4}\% = \frac{1}{2}$ of $\frac{1}{2}\%$	·853	17	
	357·481	18	
	14·299	25	interest for 2nd year.
	1·787	41	
	·893	70	
	374·461	54	
Interest at 2%	7·489	23	interest for $\frac{1}{2}$ year.
$\frac{1}{4}\% = \frac{1}{8}$ of 2%	·936	15	
$\frac{1}{8}\% = \frac{1}{2}$ of $\frac{1}{4}\%$	·468	07	
	383·355	..	

£383·355 = £383 7*s.* 1*d.*

(1) When the rate per cent. is not an integer, it is advisable to work by means of aliquot parts as shown.

(2) The interest for $\frac{1}{2}$ year at $4\frac{3}{4}$ per cent. is equivalent to the interest for 1 year at $2\frac{3}{8}$ per cent.

167. It has been assumed up to the present that interest is payable yearly. In banks, however, the books are made up and the interest calculated every half-year, while on many of the Imperial Loans interest is paid quarterly. In such cases compound interest should be calcu-

lated counting each period or term as a year, the rate of interest being reduced correspondingly.

168. When interest is payable otherwise than annually, the equivalent rate which must be paid per annum to produce the same interest is termed the *Effective Annual Rate*, *e.g.* 4% per annum paid quarterly is equivalent to 1% for four years, *i.e.* actual annual rate = 4·0604%

EXAMPLES XXIV*a*

(1) Find the compound interest on:

(*a*) £4,000 for 2 years at 3 per cent.
(*b*) £3,500 for $2\frac{1}{2}$ years at 4 per cent.
(*c*) £2,800 for 3 years at 5 per cent.
(*d*) £389 17*s*. 6*d*. for 3 years at 4 per cent.
(*e*) £857 15*s*. 9*d*. for 2 years at $3\frac{3}{4}$ per cent.
(*f*) £6,421 3*s*. 7*d*. for 3 years at $4\frac{1}{2}$ per cent.
(*g*) £2,145 11*s*. 8*d*. for $2\frac{1}{2}$ years at $4\frac{1}{3}$ per cent.

(2) Find the amount at compound interest, *payable half-yearly*, on:

(*a*) £384 18*s*. 6*d*. for 2 years at 4 per cent.
(*b*) £642 17*s*. 6*d*. for $1\frac{1}{2}$ years at 3 per cent.
(*c*) £4,231 5*s*. 11*d*. for 1 year at $2\frac{1}{2}$ per cent.

(3) Find the interest, *payable quarterly*, on:

(*a*) £5,000 for $1\frac{1}{2}$ years at 4 per cent.
(*b*) £10,000 for 1 year 3 months at 5 per cent.

(4) What is the *effective annual rate* of:

(*a*) A nominal rate of 5 per cent. per annum paid half-yearly?

(*b*) A nominal rate of 8 per cent. per annum paid quarterly?

(*c*) A nominal rate of 5 per cent. per annum paid quarterly?

169. When it is required to find the compound interest for a large number of years, the method of calculating each year's interest in the manner shown becomes very cumbersome. In these circumstances the problem is solved by means of, either—

(*a*) Compound interest tables, or
(*b*) Logarithms.

Both methods are dependent upon a knowledge of the following results:

Let £P denote the principal,
r „ „ rate per cent.,
£R „ „ amount of £1 in 1 year,
Then £R = £$(1 + \frac{r}{100})$.

The amount of £P at the end of the first year is £PR,

The amount of £P at the end of the second year is £$PR \times R$ = £PR^2,

The amount of £P at the end of the third year is £$PR^2 \times R$ = £PR^3,

and generally—

The amount of £P at the end of the nth year is £PR^n, which gives the formula $A = P\ (1 + \frac{r}{100})^n$ where £$(1 + \frac{r}{100})^n$ is the amount of £1 in n years at r% compound interest.

The values of this for different values of r and n can be obtained from compound interest tables, such as table 1, para. 170, or can be found by the use of logarithms.

When the amount of £1 has been determined, the amount for any other principal is obtained by direct multiplication.

TABLE GIVING THE AMOUNT OF £1 AT COMPOUND INTEREST

TABLE 1

Yrs.	$2\frac{1}{2}$ per cent.	3 per cent.	4 per cent.	5 per cent.
1	1·0250	1·0300	1·0400	1·0500
2	1·0506	1·0609	1·0816	1·1025
3	1·0769	1·0927	1·1249	1·1576
4	1·1038	1·1256	1·1699	1·2155
5	1·1314	1·1593	1·2167	1·2763
6	1·1597	1·1941	1·2653	1·3401
7	1·1887	1·2299	1·3159	1·4071
8	1·2184	1·2668	1·3686	1·4775
9	1·2489	1 3048	1·4233	1·5513
10	1·2801	1·3439	1·4802	1·6289

Example.—Find, by means of the above table, the compound interest on £845 17*s.* 6*d. for* 9 *years at* 3 *per cent.*

From the tables, the amount of £1 for 9 years at 3 per cent. = £1·3048.

So that the amount of £845 17*s.* 6*d.* for 9 years = £1·3048 × 845·875.

£845·875
84031
845·8750
253·7625
3·3835
6766
£1103·70..

The tables are given correct to the fourth place of decimals, so there is a possible error of £·00005. This error multiplied by 800 increases to £·04, so that there is a possible error in the result equal to 1*s.* It is useless, therefore, to work to a greater degree of accuracy than is required to give the nearest shilling.

Amount = £1,103·70	=	£1,103	14	0
Original principal	=	845	17	6
Interest	=	£257	16	6

This example is worked by logarithm tables as follows :

The formula $A = PR^n$ is expressed:

$$\log A = \log P + n \log R.$$

Substituting the values $P = £845{\cdot}875$

$R = 1{\cdot}03$

$n = 9$

we get:

$\log A$	$= \log 845{\cdot}875 + 9 \log 1{\cdot}03.$
$\therefore A$	$= £1{,}103{\cdot}678$
or Amount	= £1,103 13 7
Original principal	= £845 17 6
	£257 16 1

log 1·03	= ·0128372
	9
9 log (1·03)	= ·1155348
log 845·875	= 2·9273062
	3·0428410
antilog	= £1,103·678

EXAMPLES XXIV*b*

By means of the table in para. 169, find correct to the nearest shilling:

(1) Amount of £314 17*s.* 6*d.* for 8 years at $2\frac{1}{2}\%$
(2) ,, ,, £49 18*s.* 7*d.* ,, 10 ,, 5%
(3) ,, ,, £1,241 3*s.* 8*d.* ,, 5 ,, 4%
(4) ,, ,, £614 8*s.* 11*d.* ,, 3 ,, 3%
(5) ,, ,, £297 6*s.* 8*d.* ,, 6 ,, 4%
(6) Interest on £411 14*s.* 7*d.* ,, 9 ,, 5%
(7) ,, ,, £814 3*s.* 7*d.* for 4 years at 6% paid half-yearly.
(8) Interest on £43 6*s.* 8*d.* for 5 years at 5% paid half-yearly.
(9) Interest on £184 13*s.* 7*d.* for 3 years at 5% paid half-yearly.

170. *Construct a table showing the amount at compound interest of* £1 *to* £9 *for* 6 *years at* 3 *per cent.*

	1·00
	·03
1st year	1·03
	·0309
2nd ,,	1·0609
	·031827
3rd ,,	1·092727
	·03278181
4th ,,	1·12550881
	·03376526 43
5th ,,	1·15927407 43
	·03477822 22
6th ,,	1·19405229 65

Amount at compound interest on :

£1	for 6 years	at 3	per cent.		=	1·19405230
£2	,,	,,	,,	,,	=	2·38810460
£3	,,	,,	,,	,,	=	3·58215690
£4	,,	,,	,,	,,	=	4·77620920
£5	,,	,,	,,	,,	=	5·97026150
£6	,,	,,	,,	,,	=	7·16431380
£7	,,	,,	,,	,,	=	8·35836610
£8	,,	,,	,,	,,	=	9·55241840
£9	,,	,,	,,	,,	=	10·74647070

From the above table find the compound interest on £361 17s. 6d. for 6 years at 3 per cent.

Amount of £361·875 = £358·21569
71·64314
1·19405
·95524
·08358
·00597

Amount	=	£432·098	
Principal	=	361·875	
Interest	=	£70·223	= £70 4*s*. 5*d*.

EXAMPLES XXIVc

Construct a table similar to the above showing compound interest for 7 years at 4 per cent.

By its means find the compound interest on:

(*a*) £87 15*s*. (*c*) £317 6*s*. 8*d*.
(*b*) £450 (*d*) £421 9*s*. 11*d*.

171. *Example.—Find the principal which invested at 4 per cent. compound interest will amount in 5 years to* £500.

From the table:

Amount of £1 in 5 years at 4 per cent. = £1·2167.

∴ Since £1·2167 is the amount of £1 in 5 years at 4 per cent.,

£1 is the amount of $£\frac{1}{1·2167}$ in 5 years at 4 per cent.,

and £500 is the amount of $£\frac{500}{1·2167}$ in 5 years at 4 per cent. = £410 19*s*.

£410 19*s*. is termed the *present value* of £500 due 5 years hence at 4 per cent.

In general:

The amount of £1 in n years at r per cent = $£(1 + r)^n$

∴ Present value of £1 due in n years at r per cent. = $£\frac{1}{(1+r)^n}$

and present value of $£A$ due in n years at r per cent. = $£\frac{A}{(1+r)^n}$

It will be noticed that the *present value* of £1 under certain conditions as to time and rate per cent. is merely the reciprocal of its amount under the

same conditions. These values are themselves tabulated as in column (*b*), table II (para. 175).

Example.—Find the present value of £1,000 *due in* 7 *years' time at* 5 *per cent. compound interest* (*correct to the nearest shilling*).

From column (*b*), table 2:

P.V. of £1 due 7 years hence at 5%	= £·71068
∴ P.V. of £1,000 ,, ,, ,, ,,	= £710·68
	= £710 14*s*.

172. An annuity is a periodical payment made annually or at more frequent intervals, either for a fixed period of years or during the continuance of a given life, the payments usually being made at the end of each period.

Example.—Find the amount of an annuity of £1 *left unpaid for* 5 *years at* 5 *per cent.*

The payment due at the end of the first year, if left unpaid, accumulates at compound interest for four years, the second payment accumulates for three years, third for two years, fourth for one year, and the fifth earns no interest.

The amount of £1 in 4 years at 5% (see table II)	= £1·21551
The amount of £1 in 3 ,, ,,	= £1·15763
,, ,, £1 in 2 ,, ,,	= £1·1025
,, ,, £1 in 1 ,, ,,	= £1·05
,, ,, £1	= £1
Total amount of annuity in 5 years	= £5·52564

A person, therefore, who does not claim an annuity of £1 at 5 per cent. for 5 years is entitled to £5·52564 at the end of the period. Suppose, however, he wishes to raise money at the beginning of the period, then if the annuity is certain, he

can sell it, but will not obtain for it a sum of money greater than its present value.

But the P.V. of £5·52564 due in 5 years' time at 5 per cent., by para. 171,

$$= \frac{£5{\cdot}52564}{\text{amount of £1 in 5 years}} = \frac{£5{\cdot}52564}{£1{\cdot}27628} = £4{\cdot}32948,$$

so that the most he can expect to receive for the present sale of the annuity is £4·32948.

173. In general the amount of an unpaid annuity can be calculated as follows:

Suppose £1 to be invested for n years at r per cent. At the end of the first year £r is earned, and this is allowed to remain and accumulate. At the end of the second year another £r is earned, which is also allowed to remain, and similarly with the third, fourth, fifth, and all succeeding years, so that the original £1 is earning an annuity of £r a year which is left unpaid till the end of the nth year.

But £1 amounts to £$(1 + r)^n$ in n years at r per cent. (para. 171). So that £1 earns £$(1 + r)^n$ − £1 in n years at r per cent., and this must be the sum of the annuity of £r per year.

Since the sum of the annuity £r left unpaid for n years = £$(1 + r)^n - 1$,

$\therefore$ the sum of the annuity £1 left unpaid for n years $= £\dfrac{(1 + r)^n - 1}{r}$.

Similarly the sum of the annuity £P left unpaid for n years $= £P\left(\dfrac{(1 + r)^n - 1}{r}\right)$.

This formula can be used to calculate the amount of any annuity which is left unpaid for

a given time. In practice, however, the value of $\frac{(1+r)^n-1}{r}$ is given in tables, for different values of r and n (see column c, table II).

Note that $\frac{(1+r)^n-1}{r} = \frac{\text{compound interest on £1.}}{\text{rate per cent.}}$

Examples XXIV*d*

Using the tables in para. 169, find correct to the nearest shilling the present value of :

(1)	£1,000	due	8	years	hence at	3	per cent.
(2)	£450	,,	6	,,	,,	4	,,
(3)	£340	,,	4	,,	,,	5	,,
(4)	£1,200	,,	9	,,	,,	$2\frac{1}{2}$	,,
(5)	£860	,,	7	,,	,,	4	,,

Using table 2, column (b), para. 175, find correct to the nearest shilling the present value of :

(6)	£1,142 7*s.* 6*d.*	due	7	years	hence at	5	per cent.
(7)	£941 7*s.* 11*d.*	,,	4	,,	,,	5	,,
(8)	£842 15*s.* 6*d.*	,,	9	,,	,,	5	,,

174. Since the P.V. of £1 due n years hence at r per cent. $= £\frac{1}{(1+r)^n}$, then the P.V. of $£\frac{(1+r)^n-1}{r}$ due n years hence at r per cent. $= £\frac{(1+r)^n-1}{r(1+r)^n}$

$$\text{or} = £\frac{1}{r}\left(1-\frac{1}{(1+r)^n}\right).$$

These values are tabulated for different values of r and n (see column (d) in the table below).

175. In the following table the various columns show :

	Formulæ.
(*a*) The amount of £1 in *n* terms at 5 per cent. per term .	$(1+r)^n$
(*b*) The P.V. of £1 due in *n* terms at 5 per cent. per term .	$£\dfrac{1}{(1+r)^n}$
(*c*) The amount of an annuity of £1 per term for *n* terms at 5 per cent. per term . .	$£\dfrac{(1+r)^n-1}{r}$
(*d*) The P.V. of the above annuity	$£\dfrac{1}{r}\left(1-\dfrac{1}{(1+r)^n}\right)$

TABLE II

Years.	(*a*)	(*b*)	(*c*)	(*d*)
1	1·05	·95238	1·00	0·95238
2	1·1025	·90703	2·05	1·85941
3	1·15763	·86384	3·1525	2·72325
4	1·21551	·82270	4·31013	3·54595
5	1·27628	·78352	5·52563	4·32948
6	1·34010	·74622	6·80191	5·07570
7	1·40710	·71068	8·14201	5·78638
8	1·47746	·67684	9·54911	6·46321
9	1·55133	·64461	11·02656	7·10782
10	1·62889	·61391	12·57789	7·72173

EXAMPLES XXIV*e*

By means of table I find, correct to the nearest *s*., the amount of the following unpaid annuities :

(1) Annuity of £10 per annum for 5 years at 4%.

(2) Annuity of £10 per annum for 6 years at 3%.

(3) Annuity of £120 per annum for 4 years at 6%, payable half-yearly.

From the same table find correct to the nearest shilling the P.V. of the following annuities :

(4) £50 per annum for 8 years at 3%.

(5) £120 ,, ,, ,, 6 ,, 4%.

(6) £208 ,, ,, ,, 10 ,, $2\frac{1}{2}$%.

From table II find the P.V. of the following annuities at 5 per cent.:

(7) (*a*) £156 for 6 years. (*b*) £200 for 9 years. (*c*) £180 for 5 years.

176. *Example.—What is the P.V. (correct to the nearest £) of a leasehold house which produces an annual rent of £140 per annum clear, if 22 years of a 99 years' lease have already elapsed? Allow 5 per cent. compound interest.*

The problem is equivalent to finding the P.V. of £140 per annum for 77 years at 5 per cent.

$$= \frac{£140 \times \left(1 - \frac{1}{1{\cdot}05^{77}}\right)}{{\cdot}05}$$

$$= £2{,}800\left(1 - \frac{1}{1{\cdot}05^{77}}\right)$$

$$= £2{,}800(1 - {\cdot}0233566)$$

$$= £2{,}800({\cdot}9766434)$$

$$= £2{,}734{\cdot}601 \ldots$$

$$= £2{,}735 \text{ (correct to nearest £).}$$

$$\begin{aligned}
\log 1{\cdot}05 &= {\cdot}0211893 \\
77 \log 1{\cdot}05 &= 1{\cdot}483251 \\
& \ \ {\cdot}1483251 \\
& \ \overline{1{\cdot}6315761}
\end{aligned}$$

$$\begin{aligned}
\therefore \log \frac{1}{1{\cdot}05^{77}} &= \log 1 - 77 \log 1{\cdot}05 \\
&= 0 - 1{\cdot}6315761 \\
&= \bar{2}{\cdot}3684239 \\
\text{antilog} \quad &= {\cdot}0233566
\end{aligned}$$

In obtaining log 1·05, seven-figure tables should be used, as an error committed by the use of the more approximate four-figure tables will be multiplied 77 times in obtaining log $1{\cdot}05^{77}$. Four-figure

tables could be used to obtain the antilog, however, thus:

antilog $\bar{2}\cdot 3684 = \cdot 02335$ by four-figure tables;
and, £2,800(1 − ·02335)
= £2,800(·97665)
= £2,734·62
= £2,735 (correct to nearest £).

The final product was obtained by direct multiplication without the use of tables.

177. Machinery, buildings, etc., decrease in value from year to year, owing to wear and tear, atmospheric effects, etc., so that eventually they need replacing. To meet this depreciation of value it is usual for firms to put by a certain amount out of each year's profits, so that by the time the wasted asset is to be replaced, there will be sufficient money at hand to meet the cost of replacement without unduly disturbing the capital or the profits of the year.

Example.—If the "life" of a certain machine is ten years, what amount must be put by annually to meet the cost of replacement, given the price of a new machine is £1,000, *and the residual value of the old machine* £100? *Interest reckoned at* 5 *per cent.*

An annuity of £1 per year for 10 years at 5 per cent. amounts to $£\dfrac{1\cdot 05^{10} - 1}{\cdot 05}$.

If annuity tables are at hand, this can be obtained directly. Thus column (*b*), table II, gives the amount = £12·5779.

Otherwise working by logarithms we get:

$$\text{Amount} = £\frac{1\cdot 629 - 1}{\cdot 05}$$
$$= £62\cdot 9 \div 5$$
$$= £12\cdot 58$$

Total amount equals £1,000 − £100 = £900.

$\therefore$ Yearly rent $= £\dfrac{900}{12\cdot58} = £71\cdot55.$

$= £71$ 11*s.* correct to nearest shilling.

log 1·05	=	·0211893
10 log 1·05	=	·211893
antilog ·2119	=	1·629

log 900	=	2·9542
log 12·58	=	1·0996
		1·8546
antilog	=	71·55.

178. *Example.—A corporation borrows £50,000 to be paid back in forty annual instalments, with interest reckoned at 4 per cent. What must be the amount of each instalment?*

The instalments form an annuity, the P.V. of which = £50,000. P.V. of £1 per annum for 40 years at 4 per cent.

$$= £\frac{1}{\cdot04}\left(1 - \frac{1}{1\cdot04^{40}}\right)$$

$$= £\frac{100}{4}(1 - \cdot20825)$$

$$= £\frac{79\cdot175}{4}$$

$= £19\cdot794$

$\therefore$ Value of each instalment

$$= £\frac{50,000}{19\cdot794}$$

$= £2,526.$

log 1·04	=	·0170333
40 log 1·04	=	·681332
$\log \dfrac{1}{1\cdot04^{40}}$	=	0 − ·681332
	=	$\bar{1}\cdot318668$
antilog	=	·20825

log 50,000	=	4·6990
log 19·794	=	1·2965
		3·4025
antilog	=	2,526

179. *Example.—A building society advertises a house for £700 cash down—or for £40 cash down and £7 per month. How many such payments should be paid, reckoning twelve months to the year and compound interest at 4 per cent. per annum?*

The P.V. of an annuity $= \frac{A}{r}\left(1 - \frac{1}{(1+r)^n}\right)$

4 per cent per annum $= \frac{1}{3}$ per cent. per month.

P.V. of £7 per month at $\frac{1}{3}$ per cent.

$$= £\frac{7}{\frac{1}{3}\%}\left[1 - \frac{1}{\left(\frac{301}{300}\right)^n}\right]$$

$$= £2{,}100\left[1 - \left(\frac{300}{301}\right)^n\right]$$

This amount equals £700 − £40 = £660.

$$\therefore £660 = £2{,}100\left[1 - \left(\frac{300}{301}\right)^n\right]$$

$$\frac{660}{2100} = 1 - \left(\frac{300}{301}\right)^n$$

$$\frac{24}{35} = \left(\frac{300}{301}\right)^n$$

Taking logarithms: log 24 − log 35 = n (log 300 − log 301)

$$\therefore n = \frac{\log 35 - \log 24}{\log 301 - \log 300}$$

$$= \frac{\cdot 1639}{\cdot 0014452}$$

$$= 113{\cdot}4.$$

Number of instalments = 114.

log 301	=	2·4785665
log 300	=	2·4771213
		·0014452

log 35 = 1·5441
log 24 = 1·3802
·1639

log ·1639 = $\bar{1}$·2146
log ·0014452 = $\bar{3}$·1599
2·0547
antilog = 113·4

EXAMPLES XXIV*f*

The following logarithms will be required:

log 1·0125 = ·0053950
log 1·04 = ·0170333
log 1·05 = ·0211893
log 1·06 = ·0253059
log 1·025 = ·0107239

(1) Find the value of the following leaseholds, given:

Unexpired term of lease	Ground rent etc.	Rental	Rate per cent.
(*a*) 63 years	£450	£900	4
(*b*) 46 ,,	£80	£650	5
(*c*) 57 ,,	£50	£500	5
(*d*) 55 ,,	£90	£450	6
(*e*) 25 ,,	£80	£485	4

(2) What must be written off each year to meet the depreciation of various assets given (answer correct to the nearest £1):

Life of asset	Cost to replace	Residual value of old asset	Rate per cent.
(*a*) 10 years	£1,200	£200	4
(*b*) 25 ,,	£2,500	nil.	5
(*c*) 12 ,,	£800	£150	$2\frac{1}{2}$

(3) Find the value of the annual instalments necessary to pay off the following loans (answer correct to the nearest £1):

Amount of loan	No. of years allowed for repayment	Rate per cent.
(*a*) £40,000	25	4
(*b*) £8,500	20	5
(*c*) £75·00	10	6

(4) A furniture company advertises goods for £100 cash down, or £10 down and £3 15*s*. per month. Find the number of instalments necessary, reckoning interest at 4 per cent. Given log 301 = 2·4785665; log 300 = 2·4771213.

(5) What must be paid for each of the following annuities ? (Answer correct to the nearest £1.)

Value	Age of man	Expectation	Rate per cent.
(*a*) £100 per annum	55	15	5
(*b*) £250 ,, ,,	70	8	4
(*c*) £80 per six months	56	15	$2\frac{1}{2}$
(*d*) £200 per annum	40	25	4

(6) What premium must be paid annually to insure for £500 at the age of 50, if the present age is 28 (rate 4 per cent.) ?

(7) The population of a certain town is 264,800. What will it be in ten years' time if the number of births per 1,000 is 33, number of deaths per 1,000 is 14 ? (Answer correct to nearest hundred.) log 1·019 = ·0081742.

ANSWERS

EXAMPLES I*a*

1. (*a*) 942,839,912. (*b*) 135,241,256. (*c*) 95,832,925.
2. (*a*) 100,341,922. (*b*) 92,645,893. (*c*) 80,937,375. (*d*) 76,667,787. (*e*) 64,510,412. (*f*) 63,835,777. (*g*) 57,146,851. (*h*) 48,110,893. (*i*) 37,163,264. (*j*) 35,792,243.
Totals: 15,932,256. 28,034,712. 613,185,459. 657,152,427.

EXAMPLES I*b*

(i) 347,111. (ii) 139,087. (iii) 158,824. (iv) 117,048. (v) 265,021. (vi) 2,104,310. (vii) 2,034,188. (viii) 2,221,744. (ix) 5,600,306. (x) 396,066. (xi) 2,425,929. (xii) 5,896,713. (xiii) 2,035,297. (xiv) 1,385,340. (xv) 1,145 deficit. (xvi) 527,662. (xvii) 52,003. (xviii) 420,486. (xix) 424,635. (xx) 5,652,474. (xxi) 4,034,025. (xxii) 144,898. (xxiii) 423,172. (xxiv) 155,426. (xxv) 9,017. (xxvi) 2,002,516. (xxvii) 436,178.

EXAMPLES I*c*

1. Total, £34,150 15*s*. 11*d*.; balance, £9,785 9*s*. 10*d*.
2. Total, £2,235,853; balance, £493,696.
3. Total, £343,372 13*s*.; balance, £43,507 16*s*. 9*d*.
4. Total, £1,998,271; balance, £441,445.
5. Total, £186,042 12*s*.; balance, £17,851 1*s*. 5*d*.
6. Total, £1,243,550; balance, £51,720.

EXAMPLES II

1. (*a*) 319,475. (*b*) 1,597,375. (*c*) 7,986,875. (*d*) 39,934,375.
2. (*a*) 121,800. (*b*) 609,000. (*c*) 304,500. (*d*) 15,225,000.
3. (*a*) 991·5. (*g*) 4,957·5. (*c*) 24,787·5. (*d*) 12,393·75.
4. (*a*) 147·05. (*b*) 735·25. (*c*) 3,676·25. (*d*) 1,838·125.
5. (*a*) 15·74. (*b*) 78·7. (*c*) 39·35. (*d*) 196·75.
6. (*a*) 817,152. (*b*) 1,576,848. (*c*) 1,602,384. (*d*) 810,768. (*e*) 625,632. (*f*) 651,168.
7. (*a*) 870,753. (*b*) 3,472,521. (*c*) 4,378,244. (*d*) 2,192,619. (*e*) 346,203. (*f*) 430,131.
8. (*a*) 47,690·4. (*b*) 482·673. (*c*) 2,409·519. (*d*) 12,095·67. (*e*) 486·519. (*f*) 120,322·11.
9. (*a*) 17,537. (*b*) 15,691. (*c*) 16,614. (*d*) 13,845. (*e*) 11,999.

10. (*a*) 150,864. (*b*) 136,496. (*c*) 265,808. (*d*) 603,456. (*e*) 452,592.
11. (*a*) 38,675. (*b*) 80,325. (*c*) 92,225. (*d*) 142,800. (*e*) 166,600.
12. (*a*) 95,676. (*b*) 104,244. (*c*) 41,412. (*d*) 129,948. (*e*) 75,684.
13. (*a*) 12,560,768. (*b*) 7,357,392. (*c*) 12,988,976. (*d*) 14,130,864.
14. (*a*) 30,006,801. (*b*) 13,716,297. (*c*) 55,787,292. (*d*) 18,595,764.
15. (*a*) 855,820,725. (*b*) 3,418,158,225. (*c*) 350,186,125. (*d*) 22,206,925.
16. (*a*) 12,779. (*b*) 2,555·8. (*c*) 511·16. (*d*) 102·232.
17. (*a*) 48·72. (*b*) 9·744. (*c*) 19·488. (*d*) ·38976.
18. (*a*) 3,966. (*b*) 793·2. (*c*) 158·64. (*d*) 317·28.
19. (*a*) 58,820. (*b*) 11,764. (*c*) 2,352·8. (*d*) 4,705·6.
20. (*a*) 629,600. (*b*) 125,920. (*c*) 251,840. (*d*) 50,368.
21. (*a*) 577·5. (*b*) 545·9. (*c*) 1,021·8. (*d*) 738. (*e*) 1,476.
22. (*a*) 1,172·7. (*b*) 1,442·8. (*c*) 1,317·0. (*d*) 1,380·8. (*e*) 1,131·4.
23. (*a*) 1,182·4. (*b*) 252·7. (*c*) 126·1. (*d*) 58·6. (*e*) 52·4.
24. (*a*) 70·8. (*b*) 74·0. (*c*) 34·4. (*d*) 111·1. (*e*) 160·6.
25. (*a*) 2,566·4. (*b*) 1,003·7. (*c*) 113·1.
26. (*a*) 374·9. (*b*) 76·1. (*c*) 25·8.

EXAMPLES III*a*

1.

(*a*)	(*b*)
768,734,700	768,735,000
696,635,100	696,635,000
851,893,400	851,893,000
948,506,500	948,506,000
1,064,164,700	1,064,165,000

2. (*a*) 3·642, $\frac{1}{20000}$. (*b*) 4·587, $\frac{1}{10000}$. (*c*) 3·684, $\frac{1}{10000}$. (*d*) 2·749, $\frac{1}{15000}$. (*e*) 9·615, $\frac{1}{50000}$. (*f*) 3·413, $\frac{1}{9000}$.

EXAMPLES III*b*

1. (*a*) ·013541$\dot{6}$. (*b*) ·03958$\dot{3}$. (*c*) ·005208$\dot{3}$. (*d*) ·01041$\dot{6}$. (*e*) ·02395$\dot{8}$3. (*f*) ·047875. (*g*) ·038541$\dot{6}$. (*h*) ·015625. (*j*) ·017708$\dot{3}$. (*k*) ·03125. (*l*) ·034375. (*m*) ·02708$\dot{3}$. (*n*) ·04375. (*o*) ·0208$\dot{3}$. (*p*) ·019791$\dot{6}$.
2. (*a*) 9·311458$\dot{3}$. (*b*) ·91041$\dot{6}$. (*c*) ·621875. (*d*) 7·698958$\dot{3}$. (*e*) 3·9$\dot{6}$. (*f*) 5·575. (*g*) ·9375. (*h*) 2·784375. (*i*) 9·830208$\dot{3}$. (*j*) 3·659375. (*k*) 7·8375. (*l*) 2·21458$\dot{3}$.
3. (i) (*a*) ·014. (*b*) ·040. (*c*) ·005. (*d*) ·010. (*e*) ·024. (*f*) ·048. (*g*) ·039. (*h*) ·016. (*j*) ·018. (*k*) ·031. (*l*) ·034. (*m*) ·027. (*n*) ·044. (*o*) ·021. (*p*) ·020.
   (ii) (*a*) 9·311. (*b*) ·910. (*c*) ·622. (*d*) 7·699. (*e*) 3·967. (*f*) 5·575. (*g*) ·937. (*h*) 2·784. (*i*) 9·830. (*j*) 3·659. (*k*) 7·837. (*l*) 2·215.
4. (*a*) £3 14*s*. 10*d*. (*b*) £4 11*s*. 3*d*. (*c*) £8 19*s*. 5*d*. (*d*) £2 12*s*. 11½*d*. (*e*) £7 11*s*. 8¼*d*. (*f*) £2 7*s*. 4¼*d*. (*g*) £2 15*s*. 6¾*d*. (*h*) £3 10*s*. 7½*d*. (*i*) 16*s*. 8*d*. (*j*) 8*d*. (*k*) £1 0*s*. 0¾*d*. (*l*) £1 2*s*. 2¾*d*.

EXAMPLES IIIc

1. (a) 3·246875. (b) 6·765625. (c) ·859375. (d) 4·28125. (e) 13·840625.
2. (a) 3·247. (b) 6·766. (c) ·859. (d) 4·281. (e) 13·841.

EXAMPLES IIId

1. (a) ·625. (b) ·8125. (c) 4·3304. (d) 18·1428. (e) 3·3125. (f) 5·6518.
2. See Ex. IIIb, No. 2.
3. (a) ·8611. (b) ·9444. (c) ·6389. (d) ·4722. (e) 1·4167. (f) 6·6944. (g) 9·5. (h) ·9167. (i) ·5556.
4. (a) (i) 5·73636. (ii) 3·36818. (iii) 2·25909. (iv) ·79545. (v) ·43182.
4. (b) (i) ·50289. (ii) ·07521. (iii) ·55455. (iv) ·07045. (v) ·90331.
4. (c) (i) ·50694. (ii) ·72917. (iii) 3·11806. (iv) ·14583. (v) ·95139.

EXAMPLES IVa

1. (a) £52 10s. (b) £69. (c) £36 2s. 6d. (d) £65. (e) £77. (f) £223 4s.
2. (a) £46 11s. 10½d. (b) £32 11s. 11¾d. (c) £161 14s. 4½d.
3. (a) £278 19s. 3d. (b) £203 8s. 6½d. (c) £623 15s. 2½d.
4. (a) £108 17s. 6d. (b) £403 2s. 6d. (c) £53 18s. 1½d.
5. (a) £973 10s. 5d. (b) £1,299 7s. 6d. (c) £5,194 6s.
6. (a) £35 0s. 8¼d. (b) £140 15s. 1¾d. (c) £3 14s. 9¾d. (d) £126 13s. 8½d. (e) £6 10s. 3¼d.
7. (a) £9 5s. 2d. (b) £9 2s. 11d.
8. (a) £4,477 14s. 7d. (b) £11,424 13s. 3d. (c) £18,913 5s. 5d. (d) £15,639 1s. 2d. (e) £1,976 6s. 7d. (f) £816 4s. 10d.

EXAMPLES IVb

1. £12 5s. 10d.
2. £36 3s. 5d.
3. £13 4s. 3d.
4. £1,616 2s. 11d.
5. £265 13s. 11d.
6. £192 13s. 0d.
7. £173 15s. 11d.
8. £226 7s. 7d.
9. £21 9s. 7d.
10. £30 11s. 10½d.
11. £54 18s. 6d.
12. £17 7s. 7d.
13. £9 17s. 10d.
14. £9 0s. 1d.
15. £3 6s. 5½d.
16. £840 0s. 0d.
17. 84 cwts. 1 qr. 10 lbs.
18. 49 tons 26 lbs.
19. 1 cwt. 2 qrs. 20¼ lbs.
20. £526 15s. 5½d.
21. £4,096 11s. 10d.
22. £44 12s. 9d.
23. £16 0s. 4½d.

EXAMPLES Va

1. 18*s*., 13*s*., 27*s*.
2. 6*s*. 1½*d*., 12*s*. 6½*d*., 15*s*. 9*d*.
3. £2 4*s*. 7½*d*., £8 8*s*., £14 19*s*. 3*d*.
4. £2 6*s*. 10½*d*., £7 16*s*. 3*d*., £9 2*s*. 3½*d*.
5. £33 2*s*. 6*d*.
6. £122 11*s*. 6*d*.
7. £68 7*s*. 3*d*.
8. £24 1*s*. 6*d*.
9. See text.
10. MR. J. PURCHASER.

Bought of Merchant & Sons

£	*s*.	*d*.
2	17	0
0	16	6
2	7	3
2	5	0
1	2	6
£9	8	3

EXAMPLES V*b*

1. £7 3*s*. 9*d*.
2. £1 10*s*.

EXAMPLES V*c*

1. £221 18*s*. less £12 10*s*. 11*d*. = £209 7*s*. 1*d*.
2. £816 less £45 4*s*. = £770 16*s*.
3. £1,340 less £76 10*s*. = £1,263 10*s*.
4. £338 less £24 18*s*. = £313 2*s*.

EXAMPLES V*d*

1. £1,120.
2. Gross profit, £712. Net profit, £265.
3. Gross profit, £992. Net profit, £295.
4. Gross profit, £3,096. Net profit, £1,353.

EXAMPLES VI

1. £142,135.
2. £71,493. £82,781 12*s*.
3. £10,862 14*s*. 9*d*.
4. (*a*) 24·3. (*b*) 15·1. (*c*) 14·5. (*d*) 14·7.
5. £865,986,874.
6. 62·6°.
7. (*a*) 19*s*. 1*d*. (*b*) £298 13*s*. 1*d*. (*c*) 18*s*. 3*d*.
8. £90. £124.

	Av. cost per mile.	Av. cost per mile per ton.
9.	4·55*d.*	2·275*d.*
	4·278*d.*	2·852*d.*
	3·834*d.*	3·834*d.*

10. £1 1*s.* 9¼*d.*
11. 11,000.
12. 930 yds.

Examples VII*a*

See Examples IVb

Examples VII*b*

		1903	1912
1.	(*a*)	282.	247.
	(*b*)	209.	203.
	(*c*)	311.	356.
	(*d*)	573.	671.

2. (*a*) 6*s.* 10*d.* (*b*) 8*s.* 3*d.*
3. (*a*) £4 9*s.* 5*d.* (*b*) £3 1*s.* 9*d.* (*c*) 11*s.* 8*d.* (*d*) £1 19*s.* 9*d.*
4. (*a*) 175. (*b*) 669. (*c*) 272. (*d*) 160. (*e*) 135.
5. (*a*) 43,700,000. (*b*) 41,100,000. (*c*) 44,500,000. (*d*) 44,900,000. (*e*) 45,300,000.

Examples VIII*a*

3. 5,463 m. 4,030 m.
4. 70,000 mm. 4,320 mm.
5. 489,734 cl.
6. 30,495 dg.
7. 3·179 m.
8. 4·179547 Km.
9. 7·3965 Kg.

Examples VIII*b*

1. 3·937 in., ·3937 in., ·03937 in.
2. 10·94 yds., 109·36 yds., 1093·61 yds.
3. ·1936 pts.
4. 27·5 bus.
5. Mile is greater; 666 yds.
6. $\frac{1093}{1760} = \frac{5}{8}$ approx.
7. 35 lbs., $\frac{55}{36}$.
8. $\frac{2}{7}$ Kl.
9. 299 Kg.
10. 1,609·3.
11. Latter 59 pts.
12. 192·3 Kg.

Examples VIII*c*

2. (*a*) 43,650 sq. m. (*b*) 56·78 sq. m. (*c*) 56,000 sq. m. (*d*) 3·7364 sq. m.
3. 4046·7 sq. m.
4. (*a*) 5,718,000 cu. m. (*b*) 35·65 cu. m. (*c*) 46,500,000 cu. m. (*d*) ·417165 cu. m.
5. 1 Kg. = weight of 1 litre of water.
6. 5·37 Kg.

Examples VIII*d*

1. 671 fr. 20 c.
2. £30 6*s*. 10*d*.
3. £294 1*s*.
4. 2,016 fr. 19 c.
5. French. 5d.
6. 1*s*. 6*d*.
7. (*b*) ·72. (*c*) ·36. (*d*) ·23. (*e*) 5·43. (*f*) ·18.
8. 4·83 fr., 3·68 fr., 10·81 fr., 17·135 fr., 26·91 fr., 33·925 fr.
9. 5*s*. 9*d*., 2*s*. 6*d*., 10$\frac{1}{4}$*d*., 8*s*. 10*d*., 7$\frac{3}{4}$*d*.
10. 11$\frac{1}{2}$*d*., 1*s*. 8$\frac{1}{2}$*d*., 3*s*. 3$\frac{3}{4}$*d*., 8*s*. 7$\frac{3}{4}$*d*.
11. 1·84 fr., 4·14 fr., 1·265 fr., 8·74 fr.
12. 1·26 fr., 7·20 fr., 1·17 fr., 2·88 fr.
13. 4*s*. 0$\frac{3}{4}$*d*., 1*s*. 1$\frac{1}{2}$*d*., 9$\frac{1}{2}$*d*., 8*s*. 9$\frac{1}{4}$*d*.

Examples IX*a*

1. (*a*) 4 acres 3,630 sq. yds. (*b*) 2 acres 2,920 sq. yds. (*c*) 5 acres 4,690 sq. yds. (*d*) 1 acre 746 sq. yds.
2. (*a*) 68. (*b*) 25. (*c*) 34. (*d*) 15. (*e*) 22.
3. £17.
4. £55 4*s*.
5. £5 5*s*. 2*d*.
6. 3,744.
7. 5*s*. 5*d*.
8. 44 ; 76 sq. in.
9. £324.
10. (*a*) 600 sq. ft. (*b*) 336 sq. ft. (*c*) 11,064 sq. ft.
11. 6*d*.
12. 844.
13. 135 fr., 47 c. £4 12*s*. 5$\frac{1}{2}$*d*.
14. 385 francs.
15. 400,000 fr. (approx.).

Examples IX*b*

1. (*a*) £1 12*s*. 6*d*. (*b*) £2 3*s*. 4*d*. (*c*) £3 6*s*. (*d*) £3 7*s*. 8*d*. (*e*) £1 16*s*. 8*d*.
2. (*a*) £3 9*s*. 5*d*. (*b*) £3 7*s*. 9*d*. (*c*) £1 17*s*. 4*d*.
3. £5 9*s*. 8*d*.
4. £2 9*s*. 8*d*.
5. £5 3*s*. 11*d*.
6. £10 7*s*.
7. £10 18*s*. 5*d*.
8. £11 3*s*. 2$\frac{1}{2}$*d*.
9. Centre £5 7*s*. Border £11 5*s*.
10. 45$\frac{1}{3}$ yds.
11. £7 15*s*. 7*d*.
12. (*a*) 348 sq. in. £2 16*s*. 3$\frac{1}{2}$*d*.
13. £1 8 fl. 3 c.
14. £19 4 fl.
15. £3 6 fl. 7 c.
16. £5 2 c.

Examples IX*c*

1. 75.
2. 8$\frac{1}{3}$: £1 0*s*. 10*d*.
3. £36 13*s*. 4*d*.
4. 1,620.
5. £47 5*s*.
6. 126.

Examples IX*d*

1. 10,880. £38 1*s*. 7*d*.
2. 1$\frac{49}{51}$.
3. 6,912. 146 cubic ft.
4. 571 ft. 9 in.
5. £290 17*s*. 5$\frac{1}{2}$*d*.
6. £161 2*s*. 3*d*.
7. £87 8*s*. 8$\frac{1}{2}$*d*.

Examples IX*e*

1. 7 c. f. 577 c. in.
2. $52\frac{1}{2}$ lbs.
3. 359 lbs.
4. 5,400,000.
5. 64,821 tons.
6. 6·6 ins.
7. 531 lbs.
8. 2*s.* 8*d.*
9. 30,712,500.
10. (*a*) 75 tonnes. (*b*) $8\frac{1}{3}$. (*c*) 6 mm.
11. 26 Kg.

Examples X*a*

1. 5*s.*
2. £3 17*s.*
3. £1 7*s.*
4. $3\frac{1}{3}$ hrs.
5. 55 days.
6. £19 16*s.* 8*d.*
7. 84 days.
8. $6\frac{2}{3}$ miles.
9. $241\frac{1}{3}$ yds.
10. 8 days.
11. 16*s.*
12. £2,884 10*s.*
13. £400.
14. 8 fr. 40 c.
15. £279 17*s.* 11*d.*

Examples X*b*

1. $2\frac{7}{13}$ weeks.
2. 25 men.
3. 14 days.
4. 15 days.
5. 52 weeks.
6. 130 tons.

Examples X*c*

1. A. £100. B. £150. C. £200.
2. A. £346. B. £519. C. £865.
3. £212. £154 13*s.* 4*d.* £266 13*s.* 4*d.* £426 13*s.* 4*d.*
4. £720. £800. £520.
5. £918. £864. £756. £540.
6. £493. £667. £754. £348.
7. £12. £10. £6.

Examples XI*a*

1. 34·9%.
2. 17·2%.
3. 27·3%. 21·4%.
4. 22·5%.
5. 11·1%.
6. 3·4%.
7. 10·3%. 51·3%. 10·6%. 21·0%.
8. 25%. 5·8%.

Examples XI*c*

1. (*a*) £10 10*s.* (*b*) £6 16*s.* (*c*) £19 17*s.* 7*d.* (*d*) £38 19*s.* 8*d.* (*e*) £3 8*s.* 8*d.* (*f*) £11 19*s.* 4*d.* (*g*) £22 4*s.* 5*d.* (*h*) £3 5*s.* 9*d.* (*i*) £4 10*s.*
2. £59 8*s.*
3. 1 gall. 1 qt. 1 pt.
4. 9*s.* 7*d.*
5. £734.
6. £376.
7. 15·1% loss.
8. 3*s.* 5*d.*
9. 1*s.* $9\frac{1}{2}$*d.*
10. 91·3%.

Examples XI*d*.

1. 67·0. 67·0. 54·8. 54·7. 70·4. 73·1. 74·1. 73·1. 70·2. 59·3.
2. 13·6.
3. 14·4%. 48·6%. 70·3%. 23·3%.
4. 12·2%.
5. 6·6%.
6. 5·5%.

Examples XII*a*

1. 20% gain.
2. 25% gain.
3. 20% gain.
4. 5% loss.
5. 20% gain.
6. 5% loss.
7. £8 18*s*. 3*d*.
8. £6 6*s*.
9. £1 12*s*. 8*d*.
10. £3 4*s*. 7*d*.
11. 14*s*. 7½*d*.
12. £129 10*s*.
13. £87 8*s*.
14. £32 11*s*.
15. £1.
16. £1 10*s*.
17. £30.
18. £1.
19. £1 3*s*. 10½*d*. (approx.).
20. £2 3*s*. 1½*d*. (approx.).
21. 1*s*. 6*d*.
22. 4*s*. 4½*d*.
23. 10*s*. 10*d*.
24. £1 17*s*. 4½*d*.
25. 4½*d*.
26. 3½*d*.
27. £1 12*s*. 1*d*.
28. £4., 80%.
29. $32\frac{8}{11}$%.
30. 65%.

Examples XII*b*

1. £1,120.
2. £265.
3. £295.
4. £1,353.

Examples XIII*a*

1. £52.
2. £107 5*s*.
3. £62 10*s*.
4. £47 10*s*.
5. £242.
6. £52.
7. £21 8*s*. 11*d*.
8. £44 8*s*. 7*d*.
9. £71 10*s*. 6*d*.
10. £127 10*s*. 5*d*.
11. £27 11*s*. 5*d*.
12. £92 11*s*. 7*d*.
13. £255 15*s*. 2*d*.
14. £411 6*s*. 3*d*.
15. £471 14*s*. 9*d*.
16. £451 7*s*. 6*d*.
17. £128 18*s*. 6*d*.
18. £100 1*s*. 5*d*.

Examples XIII*b*

1. 2*s*. 5*d*.
2. 3*s*. 8*d*.
3. 7*s*. 11*d*.
4. 15*s*.
5. 18*s*. 3*d*.
6. £1 7*s*.
7. £1 18*s*. 11*d*.
8. £2 1*s*. 1*d*.
9. £3 14*s*. 9*d*.
10. £12 19*s*. 7*d*.

11. £1 8*s*. 7*d*.
12. £4 7*s*. 1*d*.
13. £6 13*s*. 11*d*.
14. £5 1*s*. 1*d*.
15. £5 2*s*. 7*d*.
16. £13 2*s*. 7*d*.
17. £2 8*s*. 5*d*.
18. 15*s*. 3*d*.

Examples XIII*c*

1. £5 15*s*. 8*d*.
2. £2 7*s*. 8*d*.
3. £1 14*s*. 6*d*.
4. £1 3*s*. 10*d*.
5. £2 16*s*. 6*d*.

Examples XIV*a*

1. £19 7*s*.
2. £318 10*s*.
3. £52 10*s*. 8*d*.
4. £25 2*s*. 6*d*.
5. Latter by £5 10*s*.
6. £26 8*s*. 9*d*.
7. 15*s*. 10*d*.
8. £56 5*s*. 5*d*.
9. £27 13*s*. 6*d*.
10. £8.
11. £280
12. £360.
13. £29 2*s*. 3*d*.
14. £15,691 0*s*. 8*d*.; £2,385,037 1*s*. 4*d*.
15. Former by £2 18*s*. 6*d*.

Examples XIV*b*

1. 12*s*.
2. 7*s*.
3. £65 4*s*. 7*d*.
4. £327 12*s*.
5. 7*s*. 11*d*.
6. £26.

Examples XIV*c*

1. £37.
2. £227 10*s*.

Examples XIV*d*

1. 1*s*. 3*d*.
2. 1*s*. $5\frac{1}{2}$*d*.
3. £19 1*s*. 3*d*.
4. £76 5*s*. 5*d*.
5. £13 4*s*. 10*d*.
6. £10 6*s*.

Examples XV*a*

1. £5; £3 18*s*. 8*d*.; £2 11*s*.
2. £7; £7 18*s*.; £19 14*s*. 8*d*.
3. £25 16*s*. 8*d*.
4. £8 2*s*. 6*d*.; £6 16*s*. 6*d*.; £5 8*s*. 10*d*.
5. £3 3*s*.
6. £16 1*s*. 9*d*.
7. £31 5*s*. 9*d*.
8. 24%.
9. Sale price £1 10*s*. less than list price.
10. £3 15*s*.
11. A. & Co., by £4 2*s*.
12. 12*s*.; £1 9*s*. 4*d*.; £1 18*s*. 8*d*.
13. 11*s*. $1\frac{1}{3}$*d*.; £1 17*s*. 6*d*.; £3 2*s*. 6*d*.
14. £22 16*s*. 4*d*.

15. 1·64.
16. £3 6s. ; £6 1s. ; £9 13s.
17. $33\frac{1}{3}$%.
18. 3d.
19. £52 13s.
20. 8%.
21. $13\frac{1}{3}$%.
22. 23%.
23. 2s. 10d.
24. 18s. 9d.

Examples XV*b*

1. 4s. 11d.
2. 3s. 10d.
3. 7s. 4d.
4. 7s. 7d.
5. £1 12s.
6. £9 7s. 3d.
7. £1 7s.
8. £273 7s. 11d.
9. 4s. 6d.
10. £323 4s. 3d.
11. £4 1s. 10d.
12. £546 16s. 7d.
13. £2 11s. 11d.
14. £2 8s. 11d.
15. £8,142 14s. 2d.
16. £2,071 0s. 6d.

Examples XVI*a*

1. £148 19s. 9d.
2. £518 13s. 5d.
3. 2,758.80 dollars.
4. 4,373.25 marks.
5. £164 10s. 9d.
6. £86 19s. 6d.
7. £1.
8. 6,916.50 francs.

Examples XVI*b*

1. 25.15 fr.
2. 50·7d.
3. 12.42 florins.
4. 26.43 fr.
5. 25.41 fr.
6. 12.39 florins.
7. 50·4d.
8. via Amsterdam.
9. £546 17s. 6d.
10. 803 dollars.

Examples XVII*a*

1. Balance, £86.
2. Balance, £90 9s. 6d.
3. Balance, £132 13s. 3d.
4. Dr. Cr.

	Princ.			Days	Interest				Princ.			Days	Interest		
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
	110			90	1	7	1		80			90		19	9
	86			78		18	4		100			32		8	9
	164			41		18	5		150			30		12	4
	276			6		4	6	Bal. of Int.					1	7	6
Bal. of Int.	1	7	6					Bal. c/f	307	7	6				
	637	7	6		3	8	4		637	7	6		3	8	4

5. Dr. Cr.

	Princ.			Days	Interest				Princ.			Days	Interest		
	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
	540			91	6	14	7		500			17	1	3	3
	130	10		74	1	6	5		25			71		4	10
	381	7	6	56	2	19	1		200			41	1	2	5
	170			36		16	9		150			31		12	9
	212			27		15	8		200			55	1	10	2
	156	2	6	12		5	1								
	11	4	6					Bal. of Int.					11	4	6
Int. in red.					1	10	2	Bal. c/f	526	4	6				
	1,601	4	6		14	7	9		1,601	4	6		14	7	9

Examples XVIII*a*

1. £87.
2. £158 8*s*.
3. £308 8*s*. 6*d*.
4. £477 10*s*. 5*d*.
5. £124 7*s*. 7*d*.
6. £26 2*s*.

Examples XVIII*b*

1. £225.
2. £137 6*s*. 8*d*.
3. £199 11*s*. 4*d*.
4. £219 5*s*. 2*d*.
5. £233 5*s*. 3*d*.
6. £334 18*s*. 9*d*.
7. £139 15*s*. 2*d*.
8. £373 6*s*. 8*d*.
9. £693 0*s*. 3*d*.
10. £177 1*s*. 8*d*.

Examples XVIII*c*

1. $5\frac{1}{3}$%.
2. $7\frac{9}{13}$%.
3. $7\frac{13}{21}$%.
4. $6\frac{16}{19}$%.
5. $5\frac{585}{688}$ or 5·85 approx.
6. 7 per cents.
7. 5 per cents.
8. $4\frac{1}{2}$ per cents.

Examples XVIII*d*

1. £3,200.
2. £3,368 19*s*. 8*d*.
3. £5,333 6*s*. 8*d*.
4. £5,029 18*s*. 10*d*.
5. £9,048 9*s*. 10*d*.
6. £2,680.
7. £167.
8. £3,200.
9. £5,090 18*s*. 2*d*.
10. £575.
11. £375 18*s*. 5*d*.
12. £1,890.
13. £894 1*s*. 6*d*.
14. £5,550.
15. £1,334 7*s*. 6*d*.
16. £1,260 ; $16\frac{1}{4}$ approx.

Examples XVIII*e*

1. $4\frac{12}{17}\%$.
2. 5%.
3. £100.
4. $5\frac{1}{2}$ approx.
5. $5\frac{15}{17}$.
6. £19; 23·1%.
7. £742 10*s*.; £882 7*s*. 1*d*.
8. Former by £22.
9. The same.
10. £44 5*s*. 2*d*.

Examples XIX

1. £50 4*s*. 1*d*.
2. £5 14*s*. 9*d*.
3. 10*d*.
4. 12*s*. 3*d*.
5. £43 12*s*. 6*d*.
6. 2*s*. $4\frac{1}{2}$*d*.
7. £32 4*s*. 10*d*.

Examples XX

1. (*a*) Total interest, 9*s*. 6*d*. Commission, 2*s*. 6*d*. (*b*) Interest on £3,478 for 1 day = 9*s*. 6*d*. (*c*) Interest on minimum balance, 6*s*.
2. Balance of interest, £1 0*s*. 10*d*. Interest on minimum balance, 16*s*. 2*d*.
3. Form as given.
4. (*a*) Balance of interest, May £1 9*s*. 8*d*.; June £2 5*s*. 1*d*. (*b*) 19*s*. 7*d*. (*c*) £1 4*s*. 9*d*.

Examples XXI*a*

1. (*a*) 168. (*b*) 99. (*c*) 99. (*d*) 1,224. (*e*) 572. (*f*) 64. (*g*) 2,700. (*h*) 10,800. (*i*) 400. (*j*) 16. (*k*) 1,000,000. (*l*) 0. (*m*) 250,000. (*n*) 170.
2. (*a*) 1,600. (*b*) 2,500. (*c*) 7,680. (*d*) 544. (*e*) 78.
3. 360.
4. (*a*) 40,401. (*b*) 39,204. (*c*) 1,214,404. (*d*) 2,700. (*e*) 14,396. (*f*) 39,996. (*g*) 1,190. (*h*) 526,400. (*i*) 999,951.
5. (*a*) 176 sq. ft. (*b*) 325 sq. ft. (*c*) 279 sq. m.
6. (*a*) 102 sq. ft. (*b*) 116 sq. ft. (*c*) 8·5 sq. m.
7. (*a*) 36. (*b*) 25. (*c*) 81. (*d*) 78. (*e*) 21. (*f*) 56. (*g*) 104. (*h*) 84.
8. (*a*) 9. (*b*) 4. (*c*) 16. (*d*) 9. (*e*) 4,500.

Examples XXI*b*

1. $\frac{11}{28}$, $\frac{33}{56}$, $\frac{11}{14}$, $\frac{55}{56}$.
2. (*a*) £287 10*s*. (*b*) £517. (*c*) £137 2*s*. 4*d*. (*d*) £25 5*s*.
3. (*a*) 1·8061. (*b*) 1·8010. (*c*) 1·2155.
4. 62,980,096.
5. 980·6 sq. ft.
6. (*a*) 3. (*b*) $6\frac{1}{4}$. (*c*) 9.
7. 37·6.
8. 368·3.
9. (*a*) 256. (*b*) 1·0609.
10. ·0204.

Examples XXII*a*

1. £54 6*s.* 3*d.*
2. 418 sq. ft.
3. (*a*) 43. (*b*) 53. (*c*) 132. (*d*) 284. (*e*) 315. (*f*) 3·4. (*g*) 9·8. (*h*) 3·45. (*i*) 7·01. (*j*) ·36.
4. (*a*) 251·7 sq. ft. (*b*) 175·6 sq. ft. (*c*) 195·9 sq. yds. (*d*) 382·2 sq. m. (*e*) 1,569 sq. yds.
5. (*a*) 26·9 ft. (*b*) 134·6 yds. (*c*) 180 yds.
6. (i) 21·9 ft. (ii) 52·9 yds. (iii) 74·5 m.
7. 13 ft.
8. 3 ac. 1218 sq. yds.; £121 18*s.* 9*d.*

Examples XXII*b*

1. (*a*) $113\frac{1}{7}$ sq. in. (*b*) $201\frac{1}{7}$ sq. in. (*c*) $28\frac{2}{7}$.
2. 3·09 in.
3. 28 in.
4. 14·3.
5. $314\frac{2}{7}$. $216\frac{6}{7}$ sq. ft.
6. £48 1*s.* 9*d.*
7. 71·65 sq. ft. 56·25 sq. ft.
8. 2,727.

Examples XXII*c*

1. 203·3.
2. 503.
3. 282·8.
4. 209·5 lbs.
5. 265·2.
6. $166\frac{2}{3}$.
7. 3,000 cubic ft.
8. ·29 ins.
9. 1·8 mm.

Examples XXII*d*

1. (*a*) 1,257 sq. cm. 4,189 cu. cm. (*b*) 5,542 sq. cm. 38·792 cu. cm. (*c*) 196350 sq. cm. 8,181,250 cu. cm.
2. 60·6%.
3. £78 11*s.*
4. 2,973.
5. 37 lbs. nearly.
6. 31 pts.
7. 5·3 pts.

Examples XXII*e*

1. $226\frac{1}{3}$ sq. ft. 10 ft. $4\frac{1}{2}$ in.
2. 1·36 pints.
3. 7·3 ins.
4. 831 sq. ft.

Examples XXIII*a*

1. 524,288.
2. 524,288.
3. 524,288.
4. 16.
5. 32.
6. 256.
7. 524,288.
8. 262,144.
9. 65,536.
10. 256.
11. 64.
12. 8.

Examples XXIII*b*

1. See Examples XXIII*a*.
2. (*a*) 32. (*b*) 262,144. (*c*) 8. (*d*) 512.

Examples XXIII*c*

(*a*) 2·371. (*b*) 1·540. (*c*) 31·62. (*d*) 1·778. (*e*) 3·162. (*f*) 1·334. (*g*) 3·162. (*h*) 1·778. (*i*) 1·540.

Examples XXIII*d*

1. (*a*) 2. (*b*) 1. (*c*) 1. (*d*) 5. (*e*) $\bar{3}$.
2. (*a*) 1·757. (*b*) $\bar{2}$·757. (*c*) 3·757. (*d*) $\bar{4}$·757.
3. (*a*) 42·15. (*b*) ·04215. (*c*) 421·5. (*d*) ·004215.
4. (*a*) $\bar{1}$·5848. (*b*) $\bar{3}$·6785. (*c*) $\bar{6}$·6785. (*d*) $\bar{7}$·3752.
5. (*a*) $\bar{3}$·0579. (*b*) 2·5426. (*c*) $\bar{5}$·1313. (*d*) 2·3575. (*e*) 4·8932. (*f*) 2·0413. (*g*) $\bar{3}$·8494. (*h*) 3·1431.
6. (*a*) 11·9143. (*b*) $\overline{10}$·3052. (*c*) $\bar{1}$·2648. (*d*) 6·3368. (*e*) 1·5456.
7. (*a*) 1·4857. (*b*) 1·1560. (*c*) $\bar{1}$·4945. (*d*) 1·7191. (*e*) 1·6518.
8. (i) ·4661646. (ii) 1·364321.
9. (i) 6·0759177. (ii) $\bar{5}$·8987764.

Examples XXIV*a*

1. (*a*) £243 12*s*. (*b*) £361 6*s*. 3*d*. (*c*) £441 7*s*. (*d*) £48 13*s*. 8*d*. (*e*) £65 10*s*. 10*d*. (*f*) £906 9*s*. 1*d*. (*g*) £240 11*s*. 8*d*.
2. (*a*) £416 13*s*. (*b*) £672 5*s*. (*c*) £4,337 15*s*.
3. (*a*) £307 12*s*. (*b*) £640 16*s*.
4. (*a*) 5·0625%. (*b*) 8·243%. (*c*) 5·095%.

Examples XXIV*b*

1. £383 13*s*.
2. £71 7*s*.
3. £1,510 3*s*.
4. £671 8*s*.
5. £376 4*s*.
6. £226 10*s*.
7. £217 4*s*.
8. £12 3*s*.
9. £29 10*s*.

EXAMPLES XXIV*c*

(*a*) £27 14*s*. 5½*d*. (*b*) £142 3*s*. 4½*d*. (*c*) £100 5*s*. 1½*d*.
(*d*) £133 3*s*. 3*d*.

EXAMPLES XXIV*d*

1. £789 8*s*.
2. £355 13*s*.
3. £279 14*s*.
4. £960 17*s*.
5. £653 11*s*.
6. £811 17*s*.
7. £774 10*s*.
8. £543 5*s*.

EXAMPLES XXIV*e*

1. £54 3*s*.
2. £64 14*s*.
3. £1,067 4*s*.
4. £351 1*s*.
5. £629.
6. £1,820 9*s*.
7. (*a*) £791 16*s*. (*b*) £1,421 11*s*. (*c*) £779 6*s*.

EXAMPLES XXIV*f*

1. (*a*) £10,299. (*b*) £10,192. (*c*) £8,442. (*d*) £5,757 (*e*) £6,326.
2. (*a*) £83. (*b*) £52. (*c*) £47.
3. (*a*) £2,561. (*b*) £682. (*c*) £1,019.
4. 25.
5. (*a*) £1,038. (*b*) £1,683. (*c*) £1,991. (*d*) £3,124.
6. £14 12*s*.
7. 319,600.

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969					3	7	11	14	18	21	25	28	32
							1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	12	16	19	22	25	29
14	1461	1492	1523	1553							3	6	9	12	15	18	21	24	28
					1584	1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903					3	6	9	11	14	17	20	23	26
							1931	1959	1987	2014	3	5	8	11	14	16	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	15	18	21	23
17	2304	2330	2355	2380	2405	2430					3	5	8	10	13	15	18	20	23
							2455	2480	2504	2529	2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3717	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

Antilogarithms.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
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·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
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·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

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